Public School Choice: An Economic Analysis

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Abstract

This paper presents a simple economic model in which to study the impact of public school choice. The model yields three findings. First, choice increases average school quality, but the increase is smaller when peer preferences are stronger and neighborhoods less equal. Second, choice increases aggregate welfare when peer preferences are weak or neighborhoods equal, but decreases it when peer preferences are strong and neighborhoods unequal. Third, choice increases the quality of schools and welfare of households in disadvantaged neighborhoods. These findings largely survive extensions to the model that incorporate Tiebout choice, school capacity constraints, and the assumption that the costs of exercising choice are negatively correlated with socio-economic status.

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1 Introduction

Public school choice programs, also known as open enrollment or intra-district choice programs, give households a free choice of public school and provide public schools with incentives to compete for students. Prominent examples include the nationwide programs introduced into the UK and New Zealand by market-oriented governments in the 1980s.[1] Other examples include programs specific to certain districts in Israel and the US.[2] Each program is different, but the defining characteristics are that parents have a free choice of school, schools must accept all applicants - at least up to some externally-determined capacity level - and school funding is proportionate to the number of students enrolled. Programs differ on how over-subscribed schools ration places (e.g., by lottery or by distance to school), whether students' transport costs are subsidized and whether schools receive additional funding for enrolling disadvantaged students.

Supporters of public school choice programs argue that by injecting market forces into the public school system, they will improve the quality of education that public schools provide. Some critics argue that these programs do not go far enough. In particular, they argue that these programs do nothing to enhance private school options and require public schools to compete within a straitjacket imposed by local and national education authorities. For example, since they cannot control the number or composition of students that they enroll, schools cannot compete by specializing in teaching particular types of students. In some settings (e.g., the UK) they are required to teach a National Curriculum and hence cannot compete on any curricula dimension. This fact helps account for the lukewarm praise that Chubb and Moe (1992) give the British reform.[3]

A different concern with public school choice programs is that households' perceptions of school quality depend on the composition of the student body as well as the efforts of school personnel (i.e., households have peer preferences). Writing about the New Zealand public school choice program, Ladd (2002b) argued that peer preferences would lead more-advantaged households to choose schools that enrolled more-advantaged students, thereby exacerbating educational inequality. Moreover, she claimed that peer preferences would blunt the incentive effects of this program, since both successful and unsuccessful schools would have limited incentives to improve quality. The concern for successful schools was that quality improvements would adversely affect the school's socioeconomic composition; the concern for unsuccessful schools was that despite quality improvements, they would be unable to attract parents.[4]

[1] See Chubb and Moe (1992) for an account of the British reform and Ladd and Fiske (2003b) for the New Zealand reform.

[2] See Lavy (2010) for Israel and Mikulecky (2013) for an overview of US programs.

[3] They argue that "The most glaring deficiency of the 1988 Education Reform Act is that it does almost nothing to liberate the supply of schools" (p.14). They concede that "Because of choice, success is now crucially dependent on pleasing parents, and the schools are doing what they can (which is not always much, given the constraints) to make themselves attractive." (p.16)

[4] She claims that "successful schools will be reluctant to expand if doing so requires lowering the average socioeconomic level of their students" (p.7), and that, since "no...educational strategy can make a school with a large proportion of disadvan-

In this paper we develop and analyze a model of public school choice. The model does not address the criticism that choice programs relax too few of the constraints facing public schools. We acknowledge that more radical programs would provide schools with very different incentives and might generate very different outcomes. But we wish to analyze public school choice programs as they operate in practice and, in particular, the extent to which competitive pressure exerted by parental choice improves school quality and household welfare, and how this process is influenced by peer preferences. As such, we constrain schools to choose a single action - effort - that influences school quality as perceived by parents, but assume that school composition is a second factor that influences this perceived quality.

More specifically, our baseline model features one community divided into two equal-sized neighborhoods, each containing one school. Households differ by their socio-economic status and one neighborhood contains more advantaged students than the other. Without public school choice, students attend their neighborhood school. With choice, households can enroll their children in either school, but face a cost of attending the non-neighborhood school. In accordance with how such programs operate, we assume schools must admit any student that wishes to enroll. Moreover, our baseline model abstracts from capacity constraints. It follows that households will choose the non-neighborhood school if the gain in expected school quality exceeds the cost. As noted above, we assume that quality depends on the efforts of school personnel and on the composition of the students enrolled. Efforts of school personnel are committed prior to enrollment decisions and are observed by households. Moreover, households correctly anticipate the enrollment decisions of their fellows. This means that they accurately predict school qualities. Schools obtain utility from the revenue that comes with enrollment and they can increase enrollment by exerting costly effort (and thereby increasing quality). This creates a game between the two schools and we study the equilibrium of this game.

Our analysis of this baseline model yields three main findings. First, public school choice increases average school quality, but the increase is smaller when peer preferences are stronger and neighborhoods less equal. The intuition is that the competition for enrollment unleashed by choice spurs schools to increase effort which increases average school quality. Equilibrium effort levels are higher, the greater the responsiveness of enrollment to effort. When peer preferences are stronger and neighborhoods less equal, enrollment is less responsive to effort. This is because enrollment decisions are more driven by peer group concerns than differences in school efforts. Second, public school choice increases aggregate welfare when peer preferences are weak or neighborhoods equal, and decreases welfare when peer preferences are strong and neighborhoods unequal. Positive welfare effects are driven by the effort increases - and hence school quality increases - that result from competition for enrollment. Negative welfare effects arise when, in equilibrium, enrollment

taged...students look effective" (p.7), these schools "have difficulty competing for students" (p.8).

decisions are driven by the preference for better peer groups. These choices are optimal from the household's perspective but wasteful from the perspective of society, since attending the non-neighborhood school is costly and one household's peer gain is another's peer loss. A negative aggregate impact arises when the wasteful effects of peer-driven choices overwhelm the benefits stemming from increased school quality. Third, public school choice always benefits the less affluent neighborhood in the sense that it increase the quality of its school and the welfare of its households. By contrast, choice harms the more affluent neighborhood when peer preferences are strong and neighborhoods less equal because it decreases the quality of its school. This decrease in quality arises when the benefits of greater effort by school personnel are offset by the costs of an inferior peer group.

Having understood the lessons of the baseline model, we analyze how they need to be modified when we incorporate additional features that seem intuitively likely to be important in determining the potential benefits from school choice. The first such feature is Tiebout choice. In the baseline model, neighborhoods are of fixed size, and thus without choice, enrollment is independent of school efforts. Accordingly, schools put in zero effort. If households choose where to live, and if these choices are influenced by expected school quality, then, provided there is some elasticity in housing supply, this process will make neighborhood size responsive to school efforts.[5] Thus, even without school choice, enrollments will be responsive to school efforts and this will provide schools with incentives. Moreover, its seems possible that introducing school choice could dampen the incentives provided by Tiebout choice. This is because school choice weakens the link between neighborhood and school attendance.

To incorporate Tiebout choice, we assume that the supply of housing in each neighborhood has some elasticity. We then let households choose where to purchase a house taking into account the implications for school attendance. One neighborhood is more desirable than the other due to some unmodeled amenity and higher socio-economic status households experience a lower disutility from paying any given housing price. This results in stratification, with the higher socio-economic status households living in the more desirable neighborhood. To keep the model tractable, it is assumed that, under school choice, households only find out the cost of using the non-neighborhood school after they have purchased a home. The model with only Tiebout choice at work confirms that schools will put in positive effort levels even without school choice. The question is how introducing public school choice in this setting impacts school qualities and household welfare. The model is sufficiently complex with both Tiebout and school choice, that we answer this question numerically. Interestingly, the lessons regarding the impact of school choice remain exactly as for the baseline

[5] More generally, what is important is that the number of school age children residing in the neighborhoods is sensitive to school efforts. Even if the supply of housing is inelastic, this will be the case if households differ by the number of school age children they have.

model.

The second feature we incorporate is capacity constraints. Intuitively, capacity constraints seem likely to blunt the incentive effects of choice. If the more popular school is up against a binding capacity constraint, then it cannot increase enrollment and thus presumably has no incentive to exert effort. Similarly, the less popular school, should realize that its students have effectively no alternative and thus it too has no incentive to exert effort. However, this discussion is premised on the capacity constraint binding and this is something which itself depends on school efforts and household enrollment decisions. Thus, it is determined in equilibrium.

Capacity constraints are incorporated into the baseline model by assuming that enrollment in either school cannot exceed a threshold larger than the neighborhood student population but less than the community student population. Our findings are intuitive: with weak peer preferences or equal neighborhoods, both schools put in effort under choice and the capacity constraints do not bind because students by and large attend their neighborhood schools. With strong peer effects and unequal neighborhoods, the capacity constraint for the school in the more affluent neighborhood binds. Accordingly, neither school puts in any effort. The capacity constraint for the advantaged school binds even though it puts in zero effort because peer effects drive households to exercise school choice. In intermediate cases, a mixed strategy equilibrium exists in which the advantaged school puts in positive effort and the disadvantaged school randomizes between zero effort and a positive amount which exceeds that exerted by the advantaged school. The capacity constraint binds only when the disadvantaged school chooses zero effort. Again, the lessons regarding the impact of school choice remain roughly as predicted by the baseline model. The only real difference arises from the fact that when peer preferences and neighborhood inequality are sufficiently strong that the capacity constraint binds, choice has no impact on school quality. Nonetheless, choice still increases the welfare of households in the less affluent neighborhood.

The final feature we incorporate is that more affluent households are better able to take advantage of school choice. This is a common assertion made in the debate on school choice and it is interesting to see how it impacts the results. We incorporate this feature into the baseline model by assuming that the costs of using the non-neighborhood school are negatively correlated with socio-economic status. Once again, the lessons of the baseline model are largely confirmed. The only differences concern school quality. First, school efforts no longer decrease as we increase the strength of peer preferences. This reflects the fact that after some level, stronger peer preferences actually increase the responsiveness of enrollment to school efforts. Second, the quality of the school in the less affluent neighborhood decreases because it is losing its high socio-economic students.

The rest of the paper is organized as follows. Section 2 discusses related literature. Section 3 analyzes the baseline model. Sections 4-6 analyze our three extensions of this model: in Section 4 we allow households to choose in which neighborhood to live (i.e., Tiebout choice); in Section 5 we allow schools to be capacity constrained; and in Section 6 we allow for the costs of attending the non-neighborhood school to be negatively correlated with socio-economic status. In Section 7 we conclude by summarizing and discussing our findings.

2 Related literature

We model a public school choice program in which households can choose any school, schools must accept all applicants, and seats in over-subscribed schools are rationed by distance or lottery.[6] As noted above, this type of program operates in Israel, New Zealand, the UK, and many school districts in the US.[7] Given their policy significance, these programs have attracted surprisingly little theoretical attention. Some research has focused on their implications for neighborhood composition, but has abstracted from their incentive effects.[8] The closest paper to ours in approach is Lee (1997). He employs a static model featuring two neighborhoods with local schools. As in our model, choice allows households to enroll their children in non-neighborhood schools but it is costly for them to do so. In his basic model, the quality of schools in each neighborhood is determined solely by the level of spending that is chosen by neighborhood residents and the focus is on how this spending changes when choice is introduced.[9] Lee's paper is complementary to ours in that he abstracts from peer preferences by assuming that school quality does not depend on student characteristics; we ignore the implications of choice for the level of school spending chosen by voters.

Empirical analysis of public school choice programs has been focused on three questions. First, when given a choice, which schools do households choose? The evidence reported by Hastings et al. (2009), Burgess et al. (2014) and others is consistent with our framework: households face a cost of attending non-neighborhood

[6] There are at least two variations of this program that we do not consider. In one, parents can only exercise choice when the relevant schools authority (e.g., the local school district) judges the school currently attended to be ineffective. For example, the No Child Left Behind Act contained such a provision. In effect, this policy imposes a minimum effort constraint on schools. We would expect this to influence schools' behavior whether the resulting sanction involved school choice or another type of penalty. As such, the pressure created is distinct from the competitive pressure generated by school choice. In another variation, some schools can make admission contingent on educational attainment (e.g., a minimum score on an admissions test). Examples of this program can be found in Boston, New York, Paris and districts in the UK. It would be interesting to model this type of program, but it would substantially complicate an already complicated analysis. At minimum, a school's strategy would involve an effort choice and an entrance test score requirement; households would be distinguished by income, switching costs and their expected entrance test score.

[7] A growing literature considers the exact mechanism by which households are allocated to schools (e.g., whether this induces households to express true preferences or behave strategically). See Pathak (2011) for a review.

[8] For example, Epple and Romano (2003) analyze the equity implications of public school choice in a rich model in which parents choose neighborhoods, choose schools and vote on taxes to support schools. School quality depends on student characteristics, but schools are passive. Avery and Pathak (2015) present a related analysis that examines how the introduction of choice impacts households' neighborhood choices. Their analysis also assumes student composition determines school quality but ignores school effort.

[9] An extension allows quality to also depend on school effort under the assumption that schools care about the number of students they have enrolled.

schools (as reflected in the revealed preference for proximity), schools can attract households by exerting effort that improves perceived quality (as reflected in the revealed preference for school average test scores) and perceived quality depends on peer composition (as reflected in the revealed preference for schools with more-advantaged students).[10] Second, how does gaining access to the first-choice school impact a child's outcomes? Exploiting the lotteries used to ration seats in the public school choice systems, Cullen et al. (2006) find little impact on academic achievement, while Hastings et al. (2009) find positive effects for the households that place more weight on academic achievement when choosing schools. We assume that households choose the school that maximizes utility. Since we do not specify what households look for in schools (e.g., test scores, a safe environment etc), this may or may not be the school that maximizes academic achievement.[11] Third, how do public school choice programs impact academic and other student outcomes? The evidence here is also mixed.[12] This is consistent with our analysis, which implies that the effects of public school choice will be setting-specific (i.e., shaped by peer preferences and neighborhood inequality).[13]

There are large theoretical literatures devoted to Tiebout choice and private school vouchers (see Epple and Nechyba (2004) and Epple and Romano (2012)). Although the institutional environments are very different, our focus on the implications of peer preferences for efficiency as well as equity is related to some of these analyses.[14] Rothstein (2006) analyzes how peer preferences can dilute the incentives provided by Tiebout choice.[15] Epple and Romano (2008) analyze a conditional private school voucher scheme that links vouchers to ability. They show that these can preserve the efficiency-enhancing effects of private school vouchers without the cream-skimming that occurs when vouchers are universal. As we discuss in the Conclusion, our model has implications for some auxiliary policies such as subsidizing the transportation costs of poorer

[10] These papers reach different conclusions as to whether preferences are heterogeneous. Hastings et al. (2009) estimate that relative to lower-SES parents, higher-SES parents have stronger preferences for school average test scores than for proximity. Burgess et al. (2014) find no statistically significant differences between the preferences for school average test scores among low- and high-SES households.

[11] It follows that parents might care about peer composition whether or not there are peer effects in the production of test scores (see Angrist (2013) for a review of the evidence). For example, they may think that more favorable peers contribute to a safer school environment.

[12] Studying a nationwide choice program in New Zealand, Ladd and Fiske (2003a) find no statistically significant correlations between the principal reports of the impact of choice on learning outcomes and principal reports of the level of local competition. Lavy (2010) evaluates an Israeli public school choice reform implemented in a single school district in Tel Aviv. He finds that relative to non-reforming control districts, a district that implemented a public school choice program enjoyed significant school productivity gains as measured by dropout rates, test scores and behavioral outcomes. Evidence from the UK is also mixed. Gibbons et al. (2008) exploit across-region variation in choice and find that choice is associated with few productivity gains. Bradley and Taylor (2002) find stronger productivity gains using a difference-in-difference approach.

[13] Interestingly, Ladd (2002a) invokes peer preferences to explain the apparently disappointing effects of public school choice in New Zealand.

[14] Models of private school vouchers share the property that households trade higher quality against higher costs when deciding which school to attend, but must consider how private schools admit students (e.g., whether or not they can turn voucher students away). Models of Tiebout choice share the property that some households must pay a (housing) cost to attend higher-quality schools, but must consider how districts make tax and spending decisions.

[15] The idea is that relative to a model without peer preferences, in which there is hierarchical sorting of wealthier households to communities with more-effective schools, peer preferences can induce coordination failures that prevent the Tiebout sorting process from rewarding school effectiveness. Rothstein (2006) uses the sorting implications of his model to assess the relative strength of households' preferences for school effectiveness and peer composition. He finds stronger evidence for sorting on peers than than for sorting on effectiveness.

students. This type of policy targeting has also been considered in the voucher context (see Epple et al. (2017) for details of these policies and Neilson (2013) for a theoretical analysis).

Our model is related to models in the industrial organization literature that deal with switching costs and network effects (Farrell and Klemperer (2007)). Whereas we consider the impacts of peer preferences, papers in that literature consider the implications of size preferences (i.e., how do two firms compete when consumers' choices depend on each firm's market share).[16] The idea is that consumers want to buy (e.g., telecommunications) products with larger market shares so that they can, for example, share the same network as friends and colleagues. An obvious and important difference is that there is no price-setting in our model. Hence while this literature finds that these size-dependent preferences can generate "fat cat" effects that soften competition between firms and increase price-cost mark-ups, it does not necessarily follow that peer preferences will blunt the incentive effects of public school choice.

3 The baseline model

The model features a single community with a population of households of size 1. The community is divided into two neighborhoods, A and B, each containing $1/2$ of the population. There are two schools serving the community, one in each neighborhood. The school in neighborhood $J \in \{A, B\}$ is referred to as school J.

Households differ in their socio-economic status. There are a continuum of types indexed by s. Types are uniformly distributed on $[-\mu, \mu]$, where $\mu > 0$, so that the average socio-economic status is 0.[17] The neighborhoods are stratified and A is the more affluent neighborhood. Thus, households of type $[0, \mu]$ live in neighborhood A, while neighborhood B is comprised of types $[-\mu, 0]$. The parameter μ measures the degree of neighborhood inequality.

Each household has a child which it must send to one of the two schools. Households care about school quality (as they perceive it) but incur a utility cost c if using the school not in their neighborhood. This cost captures the additional transaction costs arising from using the non-neighborhood school and varies across households.[18] In our baseline specification, for all household types, costs are uniformly distributed on the interval $[0, \overline{c}]$, so that the fraction of households with cost less than or equal to $c \in [0, \overline{c}]$ is $c/\overline{c}$. Later in the paper, we allow costs to be correlated (negatively) with socio-economic status.

Letting q_J denote the quality of school J, a household living in neighborhood A with cost c obtains a

[16] It would be interesting to develop industrial organization models of the impacts of peer preferences, since these characterize many industries (e.g., gym memberships, dating agencies). To our knowledge, no such analyses exist.

[17] The average socio-economic status being 0 is just a normalization and involves no loss of generality.

[18] These include any additional time taken to travel to school, additional expenses arising from higher transport costs, psychic costs resulting from loss of community, etc.

payoff q_A from using school A and a payoff $q_B - c$ from using school B. Similarly, a household living in neighborhood B with cost c obtains a payoff q_B from using school B and a payoff $q_A - c$ from using school A. The quality of school J depends on the effort it exerts and on the average socio-economic status of its children. Thus,

$$q_J = e_J + \alpha s_J, \tag{1}$$

where e_J is school J's effort, s_J is the average socio-economic status of its students, and α is a parameter measuring the importance of peer composition.

The community provides schools with a per-student payment that exceeds the costs that an additional student creates. We normalize the per student surplus to one, so that school J's payoff is given by

$$E_J - \gamma \frac{e_J^2}{2}, \tag{2}$$

where E_J denotes enrollment in school J and γ is a parameter measuring the marginal cost of effort.

The timing of the interaction between schools and households is as follows. First, the two schools simultaneously commit to their effort levels e_A and e_B. Second, knowing school effort levels, households simultaneously choose in which school to enroll their children. In making this decision, they are assumed to correctly anticipate the enrollment decisions of other households.[19] For now, we ignore capacity constraints, assuming that both schools can accommodate all students who choose to enroll. Capacity constraints will be introduced in a later section.

3.1 School choice

We are interested in how school choice impacts school quality and household welfare. Our benchmark for comparison is a no-choice policy under which households must enroll their children in their neighborhood school. Under this assumption, each school's enrollment consists of the students in its neighborhood and thus is fixed at $1/2$. Since enrollment is fixed and effort is costly, schools exert zero effort. Each school's quality is therefore determined by the average socio-economic status of its students (see (1)). Thus, school A's quality is $\alpha\mu/2$ and school B's is $-\alpha\mu/2$. It follows that a household living in neighborhood A obtains a payoff $\alpha\mu/2$ from enrolling their child in school A and a household living in neighborhood B obtains a payoff

[19] This formalization of the relationship between school efforts and household enrollment decisions is admittedly highly stylized. In reality, this is a dynamic process in which schools understand that working harder with current students may bring enrollment gains in the future. Capturing this requires a dynamic model in which prospective parents observe today's school efforts and school compositions and these observations guide tomorrow's enrollment decisions. In an earlier version of this paper (Barseghyan et al. (2014)), we present a dynamic model that captures this process. The drawback with this model is that it is significantly more complicated and less amenable to extensions. Both dynamic and static models generate similar conclusions regarding the impact of public school choice.

$-\alpha\mu/2$ from enrolling their child in school B.

To understand what happens under choice, we work backwards, first analyzing the second stage when households simultaneously choose where to enroll their children, knowing school effort levels e_A and e_B. Assume that households anticipate that the quality of school A will be higher than that of school B and let the anticipated quality differential be denoted Δq (i.e., $\Delta q = q_A - q_B$). Then, all households in neighborhood A will use school A and households in neighborhood B will use school A if their costs are less than Δq and school B otherwise. Assuming that Δq is less than or equal to $\bar{c}$, it follows that the average socio-economic status of those enrolling in school A is

$$s_A = \frac{\mu}{2}\left[\frac{1-\frac{\Delta q}{\bar{c}}}{1+\frac{\Delta q}{\bar{c}}}\right], \tag{3}$$

while that of school B's students is

$$s_B = -\frac{\mu}{2}. \tag{4}$$

Using (1), this means that, if households correctly anticipate other households' decisions, Δq must satisfy the equation

$$\Delta q = \Delta e + \frac{\alpha\mu}{1+\frac{\Delta q}{\bar{c}}}, \tag{5}$$

where Δe is the effort differential (i.e., $\Delta e = e_A - e_B$). This is a quadratic equation with solution

$$\Delta q(\Delta e) = \frac{\sqrt{(\bar{c}+\Delta e)^2 + 4\alpha\mu\bar{c}} + \Delta e - \bar{c}}{2}. \tag{6}$$

Equation (6) gives us a closed form solution for the equilibrium quality differential. The solution will lie in the interval $[0, \bar{c}]$ if $\Delta e + \alpha\mu$ is non-negative and if $\Delta e + \alpha\mu/2$ is less than or equal to $\bar{c}$.

Given this, with effort levels e_A and e_B, the two schools will anticipate enrollments of

$$E_A(\Delta e) = \frac{1}{2}\left[1 + \frac{\Delta q(\Delta e)}{\bar{c}}\right], \tag{7}$$

and

$$E_B(\Delta e) = \frac{1}{2}\left[1 - \frac{\Delta q(\Delta e)}{\bar{c}}\right]. \tag{8}$$

Accordingly, if we define an equilibrium to be a pair of effort levels (e_A^*, e_B^*) such that each school J is maximizing its payoff (2) given its rival's effort level, the equilibrium effort levels will be identical and given by

$$e_A^* = e_B^* = \frac{1}{\gamma}\left[\frac{1}{2}\frac{\Delta q'(0)}{\bar{c}}\right]. \tag{9}$$

This condition just reflects the requirement that, for each school, the marginal benefit of an increase in effort must equal the marginal cost. The marginal benefit is the resulting increase in enrollment.

Computing the derivative $\Delta q'(0)$ from (6), we find that the equilibrium effort level is e_S^* (*effort under school choice*) which is defined to equal

$$e_S^* \equiv \frac{1}{\gamma}\left[\frac{1}{4\sqrt{\overline{c}^2 + 4\overline{c}\alpha\mu}} + \frac{1}{4\overline{c}}\right]. \tag{10}$$

Notice immediately that the parameters measuring strength of peer preferences (α) and the extent of neighborhood inequality (μ) enter into this expression multiplicatively (i.e., as $\alpha\mu$). Moreover, an increase in $\alpha\mu$ reduces equilibrium effort because it reduces the responsiveness of enrollment to effort. Intuitively, when either peer preferences are strong or neighborhood inequality is high, households enrollment decisions are more driven by peer group concerns than school efforts. Given (10), the equilibrium qualities of the two schools under school choice (q_A^*, q_B^*) will be given by

$$q_A^* = e_S^* + \frac{\alpha\mu}{2}\left[\frac{1 - \frac{\Delta q(0)}{\overline{c}}}{1 + \frac{\Delta q(0)}{\overline{c}}}\right], \tag{11}$$

and

$$q_B^* = e_S^* - \frac{\alpha\mu}{2}. \tag{12}$$

This analysis suggests what equilibrium under school choice must look like. However, it stops short of proving that both schools choosing effort level e_S^* is an equilibrium. For this, we have to check that, for each school, e_S^* is a genuine best response to the other school choosing e_S^*. There are two technical issues to worry about. First, for each school, is e_S^* a global maximum in the set of effort levels that give rise to a school quality differential described by (6)? All the analysis so far establishes, is that for both schools e_S^* satisfies the first order necessary condition for maximizing each school's payoff given the other school is choosing e_S^* and the effort differential Δe is such as to generate a school quality differential given by (6). Second, for each school, does e_S^* dominate effort levels that would generate an effort differential giving rise to a negative school quality differential which would not be described by (6)? When the two schools choose effort level e_S^*, the quality differential will be positive because of school A's natural advantage stemming from its location in the more affluent neighborhood. However, if, for example, school B dramatically increased its effort level above e_S^* it could make the quality differential negative. With such an effort choice, the flow of students between schools would be reversed: households from the more affluent neighborhood A would be enrolling their children in school B. Different expressions for the quality differential and the enrollments for the two schools would apply.

In the on-line Appendix, we provide a comprehensive discussion of both issues. We clearly identify the types of deviations that might threaten the existence of equilibrium. We show that for any given values of the parameters α, μ, and γ, the effort levels described in equation (10) are indeed equilibrium effort levels if the upper bound of the cost distribution $\overline{c}$ exceeds a critical level. This critical level is given by

$$\max\left\{\alpha\mu, \sqrt[3]{\frac{\alpha\mu}{\gamma}} - \alpha\mu, \frac{1}{2\gamma\alpha\mu}, \sqrt{\frac{1}{\gamma} + (\alpha\mu)^2}\right\}. \tag{13}$$

We find the assumption that $\overline{c}$ is relatively large natural, since there will likely exist households that would not exercise choice under almost any circumstances. Nonetheless, readers who would prefer not to make such restrictions, should be reassured that in parameter ranges not satisfying this sufficient condition, we did not find any examples in which the effort levels described in equation (10) were not equilibrium effort levels. We will therefore henceforth assume that the school qualities under choice will be (11) and (12).

3.2 The impact of school choice

We are now ready to study the impact of introducing school choice on school quality and household welfare. The policy debate and the empirical literature have focused on the quality impacts of choice, both in the aggregate and across schools and communities. The focus on household welfare is more in the spirit of traditional public economics.[20] Measures of household welfare include the additional costs that households incur when they choose their non-neighborhood school, which we view as a legitimate part of the social calculus.

We begin by defining the precise quantities of interest. Recall that without choice, school A's quality is $\alpha\mu/2$ and school B's is $-\alpha\mu/2$. Using (11) and (12), the changes in the two schools' qualities are

$$dq_A = e_S^* - \alpha\mu\left(\frac{\frac{\Delta q(0)}{\overline{c}}}{1+\frac{\Delta q(0)}{\overline{c}}}\right), \tag{14}$$

and

$$dq_B = e_S^*. \tag{15}$$

[20]This said, we do not consider school payoffs in our welfare measure. This is because we see the policy problem to which choice is one possible answer as improving school performance for given levels of educational spending. Eliciting more effort from school personnel is not considered a social loss. In addition, our model of schools' payoffs is too reduced form to permit a satisfactory accounting of the surplus accruing to school personnel and stakeholders.

The enrollment-weighted average change in school quality, which we denote by dq, is

$$dq = E_A(0)q_A^* + E_B(0)q_B^* = e_S^*. \tag{16}$$

The expression in (16) is so simple because changes in school qualities resulting from student composition are zero sum and hence wash out of the analysis.

Turning to welfare, three variables are of interest: the average change in welfare of households in the two neighborhoods, which we denote dW_A and dW_B, and the average change in welfare, which we denote by dW. The welfare change from choice for households in neighborhood A is just

$$dW_A = dq_A. \tag{17}$$

This reflects the fact that, since Δq is positive, all households in neighborhood A continue to send their children to school A and hence the only impact on their welfare is how the quality of their school changes. Households in neighborhood B are more complicated because some switch to school A and some do not. The non-switchers obtain a welfare change of dq_B. Those who do switch obtain a welfare change of $q_A + \alpha\mu/2 - c = dq_A + \alpha\mu - c$. Averaging over switchers and non-switchers, we obtain

$$dW_B = \left(1 - \frac{\Delta q(0)}{\overline{c}}\right) dq_B + \int_0^{\Delta q(0)} (dq_A + \alpha\mu - c)\, \frac{dc}{\overline{c}}. \tag{18}$$

Using (17) and (18), it is straightforward to show that the average welfare gain is

$$dW = dq - \left(E_A(0) - \frac{1}{2}\right) \frac{\Delta q(0)}{2}. \tag{19}$$

This shows that the average welfare gain depends on the difference between two terms. The first is the change in school average quality, which we know from (16) is just the change in school effort. The second represents the additional costs incurred by households in neighborhood B who use school A. For choice to generate positive average welfare gains, the increase in average quality must outweigh the additional costs incurred by switching households.

We are interested in the sign and magnitude of the six variables defined in (14)-(19) and in how they change with the importance of peer preferences and the extent of neighborhood inequality. We will assume that $\overline{c} > \sqrt{6/\gamma}$ and consider the impact of varying $\alpha\mu$ from 0 to $\overline{c}$.[21] Our findings concerning the impacts of

[21] The assumption that $\overline{c} > \sqrt{6/\gamma}$ plays a role in the findings that choice can reduce quality in school A and reduce average welfare. For this to happen, the range of values of $\alpha\mu$ must be sufficiently large.

choice on school quality are summarized in:[22]

Proposition 1 i) *Choice increases average school quality, but stronger peer preferences and greater neighborhood inequality reduce the extent of the increase.* **ii)** *In the school in the more affluent neighborhood, choice increases quality when peer preferences are weak and neighborhood inequality is low, but reduces it when the product of these variables exceeds a critical level. Moreover, stronger peer preferences and greater neighborhood inequality increase the reduction.* **iii)** *In the school in the less affluent neighborhood, choice increases quality but stronger peer preferences and greater neighborhood inequality reduce the extent of the increase.*

Part i) of the Proposition describes how choice impacts average school quality. The first component is unsurprising: average school quality depends solely on school effort and schools exert zero effort without choice. The second component is more interesting. The intuition is that stronger peer preferences and greater neighborhood inequality reduce the responsiveness of enrollment to school effort and therefore lead to lower efforts.

Parts ii) and iii) of the Proposition describe how choice impacts the distribution of quality across schools. The findings reflect the fact that when peer preferences are weak and neighborhood inequality is low, few households in the disadvantaged neighborhood exercise choice in equilibrium. It follows that in both schools, choice increases school effort without changing school peer groups. As such, choice increases quality in both schools. Stronger peer preferences or greater neighborhood inequality mean that more households in the disadvantaged neighborhood exercise choice. This household switching decreases peer quality in school A and leaves it unchanged in school B. Hence for school B, stronger peer preferences and greater neighborhood inequality reduce the quality gain (because they reduce the effort increase) but the quality gain remains positive. For school A, stronger peer preferences and greater neighborhood inequality generate a larger reduction in the quality gain (because they reduce effort and peer quality) and the quality gain is negative when the product of these variables exceeds a critical level.

Turning to welfare, our findings concerning the impacts of choice on household welfare are summarized in:

Proposition 2 i) *Choice increases average welfare when peer preferences are weak and neighborhood inequality is low, but reduces it when the product of these variables exceeds a critical level.* **ii)** *In the more affluent neighborhood, choice increases welfare when peer preferences are weak and neighborhood inequality is low, but reduces it when the product of these variables exceeds a critical level.* **iii)** *In the less affluent neighborhood, choice increases welfare and stronger peer preferences and higher neighborhood inequality first decrease this welfare gain but then, at some point, increase it.*

[22] The proofs of Propositions 1 and 2 can be found in the Appendix.

Again, part i) of the Proposition describes how choice impacts average welfare, and parts ii) and iii) describe the impact on the distribution of welfare across neighborhoods. To gain intuition for part i), recall that the average welfare gain from choice is just the difference between the average quality gain and the costs incurred by the switching households (equation (19)). In the context of Proposition 1, we explained how peer preferences and neighborhood inequality impact the average quality gain. Thus, we just need to understand the switching costs. When peer preferences are weak and neighborhood inequality is low, school A's peer advantage has little attraction. As a result, few households will exercise choice and hence these costs will be low. When peer preferences are stronger and neighborhood inequality greater, school A's peer advantage will drive more households to exercise choice. These switching households obviously benefit from their decision, but their benefit comes at the expense of households in the more affluent neighborhood school and thus there is no aggregate gain. Welfare is reduced by the costs that these households incur and, for high enough peer preferences and neighborhood inequality, these costs overwhelm the benefits of higher average quality.

For part ii), since no households in neighborhood A exercise choice, the change in welfare in neighborhood A equals the change in quality in school A. Thus, the intuition underlying the welfare change in the more affluent neighborhood is just that used to explain the quality change. For part iii) and neighborhood B, the main difference between the welfare and the school quality results, is that stronger peer preferences and higher neighborhood inequality can increase the welfare gain enjoyed by neighborhood B despite decreasing the quality of school B. This is because the benefits enjoyed by those households who switch to school A outweigh the costs (of lower school quality) experienced by those do not.

Figure 1 illustrates the theoretical results established in Propositions 1 and 2. The Figure assumes that $\mu = 1$, $\gamma = 2$, and $\bar{c} = 2.5$, and illustrates how the impact of public school choice varies as α ranges from 0 to 2.[23] In each panel, α is measured on the horizontal axis and the outcome variable of interest on the vertical. The first panel illustrates how enrollment in the two schools changes as α increases and shows that enrollment is higher in the advantaged school with stronger peer effects. The second panel illustrates the change in school effort created by choice and shows that it is positive but decreasing in α. The third panel illustrates the change in school quality created by choice, showing that the average change is positive, but the advantaged school experiences a decrease in quality when α is high. The fourth panel illustrates the change in welfare, showing that citizens in both neighborhoods experience an increase in welfare when α is small, but only citizens in the disadvantaged neighborhood benefit when α is high. Moreover, the change in aggregate welfare turns negative for α high enough.

[23] Obviously, equivalent results are obtained by fixing α equal to 1 and letting μ vary from 0 to 2.

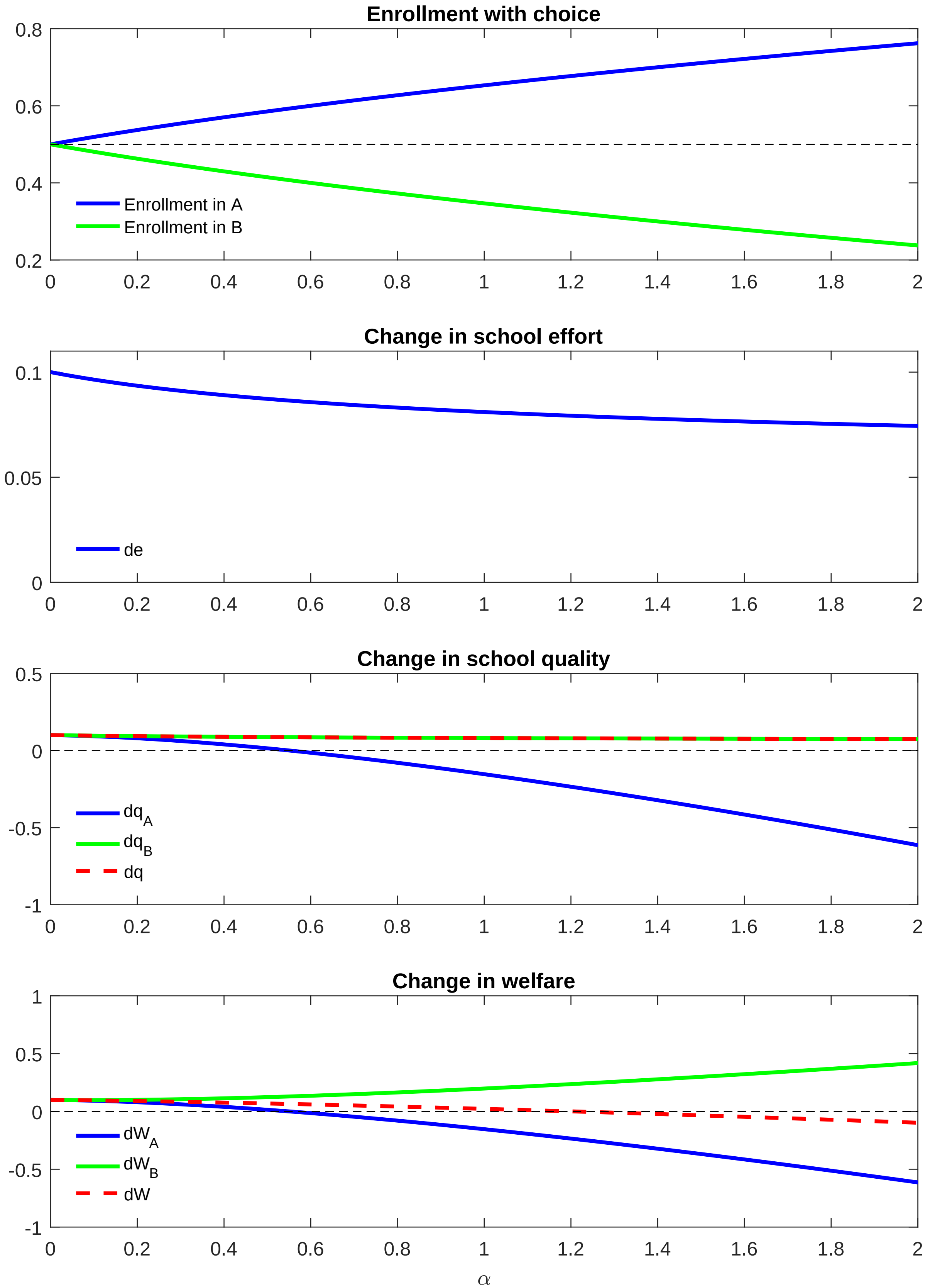

Figure 1: The impact of school choice in the basic model

4 Tiebout choice

A limitation of the baseline model, is that it by assuming that the allocation of households across neighborhoods is exogenous and independent of school quality, it ignores the incentives created by Tiebout choice. If households choose the neighborhood in which they live taking into account the quality of its school and there is some elasticity in housing supply, then enrollment will be sensitive to school efforts even when households must use their neighborhood schools. When school payoffs are increasing in enrollments, this provides schools with an incentive to provide effort even without school choice. Moreover, intuitively, it seems possible that introducing school choice could undermine the incentives created by Tiebout choice and in this way might possibly backfire.

Given this limitation, it is desirable to extend the analysis to incorporate Tiebout choice. We do this in the simplest possible way by adding an initial stage to the model in which households choose in which neighborhood to live. Thus, we continue to assume there are a continuum of household types uniformly distributed on $[-\mu, \mu]$, but now assume that each household must choose in which neighborhood to buy a house. Houses are identical within neighborhoods. Neighborhood A is more attractive than B, perhaps because it has better houses or superior amenities. This difference in neighborhoods is captured by assuming that a household of socio-economic status s who buys a house in neighborhood A obtains a housing-related payoff of $b - (\xi - s)P_A$ if he buys a house in neighborhood A and a payoff of $-(\xi - s)P_B$ if he buys in neighborhood B. Here P_J is the price of houses in neighborhood J and b is a positive parameter capturing how much more attractive neighborhood A is than neighborhood B. The parameter ξ is greater than μ, so that $\xi - s$ is positive for all types. This formulation implies that higher socio-economic status types incur a lower disutility from paying for a house which captures the idea that they have more resources and hence a lower value of the dollars left-over after purchasing a house. Higher values of ξ make households more sensitive to house price differences and thus reduce the relative demand for houses in the more expensive neighborhood.

The supply of housing in neighborhood J is assumed to be

$$S_J(P_J) = \beta + \delta_J P_J, \tag{20}$$

where $\beta \in (0, 1/2)$ and $0 < \delta_A < \delta_B$. A higher value of β makes housing supply in both neighborhoods more inelastic at any price, while a higher value of δ_J makes housing supply in neighborhood J more elastic. Assuming supply is more responsive in neighborhood B is necessary to ensure that the two neighborhoods end up equally sized in equilibrium despite the fact that neighborhood A is more desirable.

We continue to assume that, with school choice, if households use the non-neighborhood school, they incur a utility cost c and that this cost is uniformly distributed on the interval $[0, \overline{c}]$. Moreover, we further assume that at the time households choose which neighborhood to live, they are uncertain exactly what the cost of using the non-neighborhood school would be. All they know is the distribution. This assumption means that at the time they choose neighborhoods, households of the same socio-economic status will all behave in the same way, which substantially simplifies the analysis.

Introducing Tiebout choice in this way, means that there is an additional factor to be determined in equilibrium. This is the allocation of households across the two neighborhoods. Under the assumption that the neighborhoods are stratified and that, because it is more attractive, neighborhood A is more affluent, this allocation can be described by the fraction of households living in neighborhood A, which we denote x. The fraction living in neighborhood B is then $1 - x$. Given stratification, neighborhood A will consist of those households with socio-economic status $[\mu(1 - 2x), \mu]$ and neighborhood B will consist of those households with socio-economic status $[-\mu, \mu(1 - 2x)]$. Given x, the workings of the rest of the model are exactly as described in the previous section except that the neighborhood populations are described by the intervals $[\mu(1 - 2x), \mu]$ and $[-\mu, \mu(1 - 2x)]$ as opposed to $[0, \mu]$ and $[-\mu, 0]$. Accordingly, the key task in solving the extended model is to characterize the equilibrium size of neighborhood A.

4.1 Solving the model with Tiebout choice

4.1.1 Without school choice

We first solve the model without school choice. This is necessary to provide the benchmark with which to compare school choice. It is also relatively simple and provides insight into the solution of the more general case.

A household of type s buying a house in neighborhood A obtains a payoff of

$$q_A + b - (\xi - s)P_A, \tag{21}$$

while if it buys in neighborhood B, it obtains a payoff

$$q_B - (\xi - s)P_B. \tag{22}$$

Comparing these payoffs and letting ΔP denote the housing price differential $P_A - P_B$, we see that the socio-economic status of the indifferent type (i.e., the type who gets the same payoff from buying in either neighborhood) is given by $\xi - (\Delta q + b)/\Delta P$. Assuming that the school quality differential is non-negative, neighborhood A will consist of all those households with socio-economic status higher than the indifferent type. Accordingly, the size of neighborhood A is given by

$$x = \frac{\mu - \left(\xi - \frac{\Delta q + b}{\Delta P}\right)}{2\mu}. \tag{23}$$

We can make further progress by tying down the housing price differential ΔP and the school quality differential Δq. Given (20), if the size of neighborhood A is x, the house prices in the two neighborhoods must be $(x - \beta)/\delta_A$ and $(1 - x - \beta)/\delta_B$. The price differential is therefore given by

$$\Delta P = \frac{(\delta_A + \delta_B)\, x - (\beta\,(\delta_B - \delta_A) + \delta_A)}{\delta_A \delta_B}. \tag{24}$$

Turning to the quality differential, given that the indifferent type is $\xi - (\Delta q + b)/\Delta P$, the average socio-economic status of those living in neighborhood A is $[\mu + \xi - (\Delta q + b)/\Delta P]/2$ and the average socio-economic status of those living in neighborhood B is $[-\mu + \xi - (\Delta q + b)/\Delta P]/2$. Using (1), it follows that the school quality differential is

$$\Delta q = \Delta e + \alpha\mu. \tag{25}$$

This is particularly simple, since it is independent of the allocation of households across neighborhoods. Substituting (24) and (25) into (23), yields a quadratic equation in the equilibrium size of neighborhood A. This has solution

$$x(\Delta e) = \frac{\left[\begin{array}{c} \sqrt{\left[(\delta_A + \delta_B)(\xi - \mu) + 2\mu\,(\beta\,(\delta_B - \delta_A) + \delta_A)\right]^2 + 8\mu\,(\delta_B + \delta_A)(\Delta e + \alpha\mu + b)\delta_A\delta_B} \\ +2\mu\,(\beta\,(\delta_B - \delta_A) + \delta_A) - (\delta_A + \delta_B)(\xi - \mu) \end{array}\right]}{4\mu\,(\delta_B + \delta_A)}. \tag{26}$$

Equation (26) provides a closed form solution for the equilibrium size of neighborhood A for any given school effort levels.

Given this, with effort levels e_A and e_B, the two schools will anticipate enrollments of $x(\Delta e)$ and $1 - x(\Delta e)$ respectively. Defining equilibrium as in the previous section, we find that the equilibrium effort levels are identical and satisfy

$$e_A^* = e_B^* = \frac{1}{\gamma} x'(0). \tag{27}$$

Notice that equilibrium effort levels are positive, confirming the intuitive idea that Tiebout choice provides schools with incentives to exert effort. Compared with (9), the impact on enrollment is coming from the expansion of the size of the neighborhood rather than the increase in enrollment from the other neighborhood.

Computing the derivative in question, we find that the equilibrium effort level with Tiebout choice but no school choice is e_T^* (*effort under Tiebout choice*) which is defined by

$$e_T^* \equiv \frac{1}{\gamma}\left[\frac{\delta_A\delta_B}{\sqrt{\left[(\delta_A+\delta_B)(\xi-\mu)+2\mu\left(\beta\left(\delta_B-\delta_A\right)+\delta_A\right)\right]^2+8\mu\left(\delta_B+\delta_A\right)(\alpha\mu+b)\delta_A\delta_B}}\right]. \tag{28}$$

Effort is decreasing in the importance of peer effects (α) and the natural advantage of neighborhood A (b). Effort is also decreasing in β, reflecting the idea that more inelastic housing supplies dampens incentives. Finally, effort is decreasing in ξ. Recall that higher values of ξ influence the demand side of the model, by making households more price sensitive. This increases the natural advantage of the cheaper neighborhood, and thus makes neighborhood demand less responsive to school quality differences.

A natural question to ask is how the incentives provided by Tiebout choice compare with those provided by school choice. This is a difficult question to answer, because they are driven by completely driven forces. Tiebout choice incentives are limited by the elasticity of housing supply and the substitutability of the neighborhoods (which is measured by b). School choice incentives are limited by the willingness of households to choose a non-neighborhood school (which is measured by $\bar{c}$). It is possible to choose these parameters in such a way as to make the difference between e_S^* and e_T^* either positive or negative. It is notable that both efforts are reduced by stronger peer preferences. In the Tiebout case, this is because stronger peer preferences reduce the substitutability of the two neighborhoods.

To provide a benchmark with which to compare school choice, we will assume that the parameters are such that the equilibrium allocation without school choice is such that the two neighborhoods are equally sized (i.e., $x(0) = 1/2$). In the on-line Appendix, it is shown that this requires that

$$\xi = \frac{2(\alpha\mu+b)\delta_A\delta_B}{(1-2\beta)\left(\delta_B-\delta_A\right)}. \tag{29}$$

Under this condition, the qualities of the two schools are given by

$$q_A = e_T^* + \frac{\alpha\mu}{2}, \tag{30}$$

and

$$q_B = e_T^* - \frac{\alpha\mu}{2}. \tag{31}$$

4.1.2 With school choice

If school choice is possible, the payoff from buying a house in neighborhood A remains as described in (21). The payoff from buying a house in neighborhood B is changed because a household will choose to exercise choice if its cost c is less than Δq. Under the assumption that the household does not know its cost at the time of the location decision, its expected payoff from buying a house in neighborhood B is given by

$$\int_0^{\Delta q} (q_A - c)\,\frac{dc}{\overline{c}} + \int_{\Delta q}^{\overline{c}} q_B \frac{dc}{\overline{c}} - (\xi - s)P_B. \tag{32}$$

Comparing (21) and (32), we see that the socio-economic status of the indifferent type is given by $\xi - ((1 - \frac{\Delta q}{2\overline{c}})\Delta q + b)/\Delta P$. Since neighborhood A will consist of all those households with socio-economic status higher than the indifferent type, its size is given by

$$x = \frac{\mu - \left(\xi - \frac{(1-\frac{\Delta q}{2\overline{c}})\Delta q + b}{\Delta P}\right)}{2\mu}. \tag{33}$$

Using (24) to substitute in for the price differential, we find that the equilibrium size of neighborhood A, given a school quality differential Δq, is

$$x(\Delta q) = \frac{\left[\begin{array}{c}\sqrt{\left[(\delta_A+\delta_B)(\xi-\mu)+2\mu\left(\beta\left(\delta_B-\delta_A\right)+\delta_A\right)\right]^2+8\mu\left(\delta_B+\delta_A\right)((1-\frac{\Delta q}{2\overline{c}})\Delta q+b)\delta_A\delta_B} \\ +2\mu\left(\beta\left(\delta_B-\delta_A\right)+\delta_A\right)-\left(\delta_A+\delta_B\right)(\xi-\mu)\end{array}\right]}{4\mu\left(\delta_B+\delta_A\right)}. \tag{34}$$

Turning to the determinants of the school quality differential, following the same steps that led to (5), while recognizing that the neighborhood populations are described by the intervals $[\mu(1-2x), \mu]$ and $[-\mu, \mu(1-2x)]$ as opposed to $[0, \mu]$ and $[-\mu, 0]$, we find that, given x and Δe, the quality differential is

$$\Delta q(x, \Delta e) = \frac{\sqrt{(x\overline{c} + \Delta e(1-x))^2 + 4\alpha\mu\overline{c}x(1-x)} - (x\overline{c} - \Delta e(1-x))}{2(1-x)} \tag{35}$$

Given Δe, the equilibrium size of neighborhood A and school quality differential $x^*(\Delta e)$ and $\Delta q^*(\Delta e)$ are implicitly defined by the system of equations

$$\begin{aligned} x^* &= x(\Delta q^*) \\ \Delta q^* &= \Delta q(x^*, \Delta e) \end{aligned} \tag{36}$$

It is straightforward to show that there exists a solution to this system of equations for all Δe in the relevant range (see the on-line Appendix). A sufficient condition for uniqueness is that at any solution $(x^*, \Delta q^*)$ it is the case that the product $x'(\Delta q^*) \cdot \partial \Delta q(x^*, \Delta e)/\partial x$ is less than 1. While it is difficult to find simple conditions on the primitives to guarantee this condition holds, it is easily satisfied in our simulations. Thus, we will assume it is true in what follows.

Given all this, with effort levels e_A and e_B, the two schools will anticipate enrollments of

$$E_A(\Delta e) = x^*(\Delta e) + \frac{\Delta q^*(\Delta e)}{\overline{c}} \left(1 - x^*(\Delta e)\right), \tag{37}$$

and

$$E_B(\Delta e) = \left(1 - \frac{\Delta q^*(\Delta e)}{\overline{c}}\right) \left(1 - x^*(\Delta e)\right). \tag{38}$$

Defining equilibrium in the usual way, we see that the equilibrium effort levels are identical and satisfy

$$e_A^* = e_B^* = \frac{1}{\gamma} \left[\left(1 - \frac{\Delta q^*(0)}{\overline{c}}\right) \frac{dx^*(0)}{d\Delta e} + \left(1 - x^*(0)\right) \frac{\frac{d\Delta q^*(0)}{d\Delta e}}{\overline{c}} \right]. \tag{39}$$

Comparing this with (9) and (27), reveals the dual incentive effects at work with Tiebout and school choice. The first term inside the square brackets reflects the incentive effect due to Tiebout choice and the second that coming from school choice. Relative to (27), the incentive effect arising from changing neighborhood size, is dampened by the possibility of school choice. This is reflected by the multiplicative term $1 - \Delta q^*(0)/\overline{c}$. Of course, the impact of a change in the effort differential on neighborhood size may differ from that without school choice, so this does not necessarily imply the incentive effect is weaker. Relative to (9), the incentive effect arising from school choice now depends on the equilibrium size of neighborhood B (the size was equal to 1/2 in the basic model).

Further progress can be made by finding the derivatives of the system (36) (see the on-line Appendix). Calculating these and substituting in the resulting expressions into (39), we see that the equilibrium effort level is given by e_{ST}^* (*effort with school and Tiebout choice*) which is defined by

$$e_{ST}^* \equiv \frac{1}{\gamma} \left[\left(1 - \frac{\Delta q^*(0)}{\overline{c}}\right) \left(\frac{x'(\Delta q^*(0)) \frac{\partial \Delta q(x^*(0),0)}{\partial \Delta e}}{1 - x'(\Delta q^*(0)) \frac{\partial \Delta q(x^*(0),0)}{\partial x}} \right) + \left(1 - x^*(0)\right) \left(\frac{\frac{\partial \Delta q(x^*(0),0)}{\partial \Delta e}}{\overline{c} \left(1 - x'(\Delta q^*(0)) \frac{\partial \Delta q(x^*(0),0)}{\partial x}\right)} \right) \right] \tag{40}$$

The derivatives in this expression can all be calculated from (34) and (35), but the resulting expressions are cumbersome so we avoid that here. Given that neighborhood A will consist of types $[\mu(1 - 2x^*(0)), \mu]$ and neighborhood B will consist of types $[-\mu, \mu(1-2x^*(0))]$, the equilibrium school qualities will be given by

$$q_A^* = e_{ST}^* + \alpha\mu \left[\frac{x^*(0)(1-x^*(0))\left(1-\frac{\Delta q^*(0)}{\bar{c}}\right)}{x^*(0)+(1-x^*(0))\frac{\Delta q^*(0)}{\bar{c}}} \right], \tag{41}$$

and

$$q_B^* = e_{ST}^* - \alpha\mu x^*(0). \tag{42}$$

4.2 The impact of school choice with Tiebout choice

We now want to revisit the impact of school choice in the model with Tiebout choice. The benchmark allocation for comparison is that in which, prior to school choice, the two neighborhoods are equally sized. This requires that the parameters satisfy (29) and leads to school quality levels described by (30) and (31). As in Section 3, we are interested in how choice impacts school quality and household welfare. Also of interest in this setting is how choice impacts the size of the neighborhoods and housing prices. Given the complexity of the model with both Tiebout and school choice, we explore the impact of choice numerically.

Figure 2 illustrates the impact of school choice in the model with Tiebout choice. It shows how the impact depends on the importance of peer effects α. We maintain the parameter choices of the baseline model that underlie Figure 1. For the housing market parameters, we assume that $\beta = 0.25$, $\delta_A = 0.1$, $\delta_B = 0.2$, and $b = 1.5$. The housing demand-side parameter ξ adjusts to maintain equality (29) as we change α. This keeps the neighborhoods equal sized in the initial allocation with Tiebout choice alone. Welfare calculations now also take into account the surplus of housing suppliers in each neighborhood. This is done to abstract from welfare gains or losses to households that arise from choice changing housing prices. The on-line Appendix describes the formulas used to compute welfare.

The first four panels in Figure 2 replicate those in Figure 1. Strikingly, they look almost exactly the same.[24] Accordingly, the conclusions of the baseline model appear robust to allowing Tiebout choice. The final two panels describe the change in neighborhood size and the change in house prices created by choice. As expected, choice leads households to substitute to the cheaper neighborhood when peer preferences are strong. They may be able to access school A without paying the higher price of housing in neighborhood A. This raises the price of housing in neighborhood B and reduces that in neighborhood A.

One point which warrants further discussion is the change in school effort created by school choice. With Tiebout choice, school effort will already be positive without school choice (see (28)). Moreover, intuitively,

[24]This is not a reflection of the particular housing supply parameters we have chosen. Making housing supply more elastic and thus strengthening Tiebout forces, does not appreciably impact the Figure.

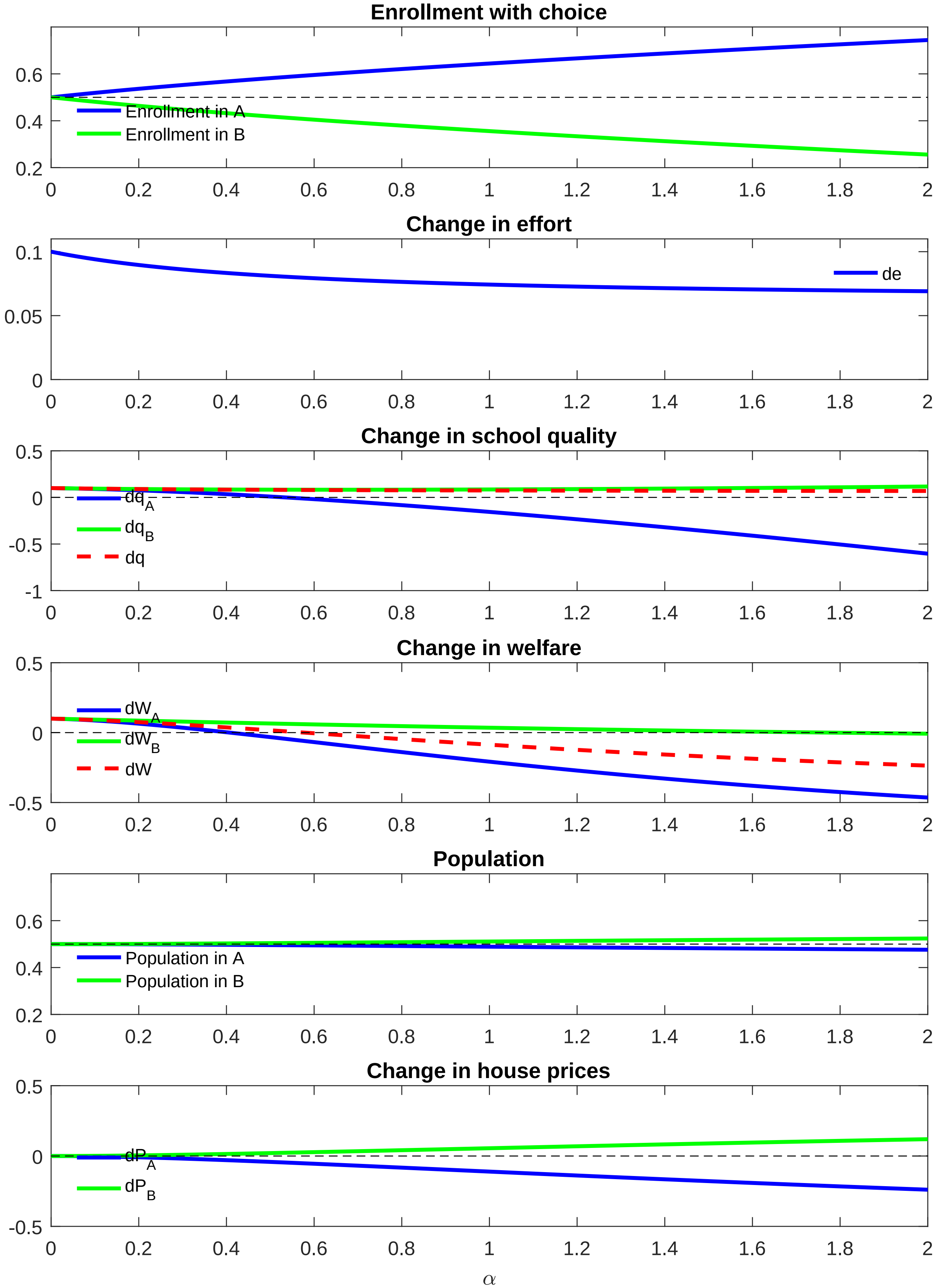

Figure 2: The impact of school choice in the model with Tiebout choice

it seems likely that school choice will dampen the incentives created by Tiebout choice. Thus, it seems that the change in school effort created by school choice should be lower with Tiebout choice than without. While this is hard to see comparing the second panels of Figure 1 and Figure 2, it is in fact the case. The only exception is when there are no peer effects ($\alpha = 0$) when the change in school effort is the same with or without Tiebout choice. Why does school choice dampen the incentives created by Tiebout choice in general, but not in the absence of peer preferences? The answer is that in general, school choice implies that a fraction of students are expected to attend the non-neighborhood school. This weakens the (Tiebout) incentives to attract households to the neighborhood. Absent peer preferences however, school choice does not induce any households to attend the non-neighborhood school (see the top panel of Figure 1). For this reason, absent peer preferences but not generally, school choice does not dampen the incentives created by Tiebout choice.[25]

5 Capacity constraints

A further limitation of the baseline model, is that it assumes that schools enroll all students who wish to attend. This ignores the possibility of capacity constraints. Intuitively, when binding, capacity constraints seem likely to reduce equilibrium efforts under choice since, once at capacity, a school has no incentive to exert further effort. In particular, therefore, if the school in the more affluent neighborhood can attract sufficient enrollees to hit its capacity constraint without exerting effort, it would seem to have no incentive to do so. Similarly, if the enrollees in the school in the less affluent neighborhood have no means of escape, then this school also has no incentive to put in effort to keep them.

To analyze capacity constraints, we return to the baseline model of Section 3 and extend it by assuming that each school faces a capacity constraint on enrollment of $\overline{E}$. We assume this maximal capacity exceeds the enrollment that each school would have with no-choice, so that $\overline{E}$ exceeds 1/2. We further assume that each school is required to enroll all students from its own neighborhood who wish to enroll. If there is excess demand from students from the other neighborhood, then the available slots are allocated randomly among those wishing to enroll.

To understand what happens, consider the second stage when households simultaneously choose in which school to enroll their children knowing school effort levels e_A and e_B. If households anticipate that the quality of school A will be higher than that of school B, all households in neighborhood A will use school A and households in neighborhood B will try to enroll their children in school A if their costs are less than Δq. The fraction of these households who will be successful, is given by $\min\{1, (\overline{E} - 1/2)/\Delta q/2\overline{c}\}$. It follows that the

[25]Formally, the finding is that when $\alpha = 0$, it is the case that $e^*_{ST} - e^*_T = e^*_S$. Using (9), (27), and (39), it is easy to verify that this will be the case if $dx^*(0)/d\Delta e$ equals $x'(0)$ and $d\Delta q^*(0)/d\Delta e$ equals $\Delta q'(0)$. Both equalities hold when $\alpha = 0$.

average socio-economic status of those enrolling in school A is

$$s_A = \frac{\mu}{2}\left[\frac{1 - \frac{\Delta q}{\bar{c}}\min\{1, \frac{\overline{E}-\frac{1}{2}}{\frac{\Delta q}{2\bar{c}}}\}}{1 + \frac{\Delta q}{\bar{c}}\min\{1, \frac{\overline{E}-\frac{1}{2}}{\frac{\Delta q}{2\bar{c}}}\}}\right], \tag{43}$$

while that in school B is

$$s_B = -\frac{\mu}{2}. \tag{44}$$

Using (1), this means that, if households correctly anticipate the decisions of other households, Δq must satisfy the equation

$$\Delta q = \Delta e + \alpha\left(\frac{\mu}{1 + \frac{\Delta q}{\bar{c}}\min\{1, \frac{\overline{E}-\frac{1}{2}}{\frac{\Delta q}{2\bar{c}}}\}}\right). \tag{45}$$

This has solution

$$\Delta q(\Delta e) = \begin{cases} \frac{\sqrt{(\bar{c}+\Delta e)^2 + 4\alpha\mu\bar{c}} + \Delta e - \bar{c}}{2} \text{ if } \Delta e \leq \left(2\overline{E}-1\right)\bar{c} - \frac{\alpha\mu}{2\overline{E}} \\ \Delta e + \frac{\alpha\mu}{2\overline{E}} \text{ if } \Delta e > \left(2\overline{E}-1\right)\bar{c} - \frac{\alpha\mu}{2\overline{E}} \end{cases}. \tag{46}$$

Equation (46) provides the equilibrium quality differential associated with any pair of effort levels. This will be the same as in the basic model if Δe is less than $\left(2\overline{E}-1\right)\bar{c} - \alpha\mu/2\overline{E}$ in which case the capacity constraint is not binding. If this condition is not satisfied, then the capacity constraint binds and the equilibrium quality differential takes a simpler form.

Given this, with effort levels e_A and e_B, the two schools will anticipate enrollments of

$$E_A(\Delta e) = \frac{1}{2}\left[1 + \frac{\Delta q(\Delta e)}{\bar{c}}\max\{1, \frac{\overline{E}-\frac{1}{2}}{\frac{\Delta q(\Delta e)}{2\bar{c}}}\}\right], \tag{47}$$

and

$$E_B(\Delta e) = \frac{1}{2}\left[1 - \frac{\Delta q(\Delta e)}{\bar{c}}\max\{1, \frac{\overline{E}-\frac{1}{2}}{\frac{\Delta q(\Delta e)}{2\bar{c}}}\}\right]. \tag{48}$$

Attempting to characterize equilibrium effort levels by taking first order conditions is no longer appropriate, because school payoff functions are not differentiable at the point at which the capacity constraint first binds. Nonetheless, it is straightforward to think through the forces shaping the schools' effort decisions. Recall first that, in the equilibrium of the basic model, the two schools put in the same effort levels and hence Δe is equal to zero. Accordingly, if it is the case that $\left(2\overline{E}-1\right)\bar{c}$ is greater than $\alpha\mu/2\overline{E}$, then the capacity constraint is not binding at the equilibrium of the basic model. However, this does not imply that the equilibrium of the basic model remains an equilibrium since the capacity constraint changes the enrollment consequences of deviations. In particular, if school B deviates to zero effort, its enrollment cannot fall below $1 - \bar{E}$. It could be that putting in zero effort and obtaining an enrollment of $1 - \bar{E}$ students dominates the payoff from matching the effort level (10) chosen by school A. When this is the case, then the equilibrium of the basic model is no longer an equilibrium.

More generally, when $\left(2\overline{E} - 1\right)\bar{c}$ is smaller than $\alpha\mu/2\overline{E}$, the equilibrium of the basic model can never be an equilibrium. If it were, then the capacity constraint would be binding and school A could reduce its effort marginally with no detrimental impact on enrollment. The only impact would be to reduce the excess demand. The same logic implies that in any equilibrium in which the capacity constraint is binding, school A must be putting in zero effort.

Is it possible that there exist equilibria in which the capacity constraint binds, school B puts in positive effort, and school A puts in zero effort? The answer is no. If the capacity constraint binds, school B can reduce its effort and have no impact on its enrollment. All that happens is the excess demand for school A will increase. It follows that the only possible equilibrium in which the capacity constraint binds for sure is that both schools put in zero effort. A necessary condition for this to exist is that $\left(2\overline{E} - 1\right)\bar{c}$ is smaller than $\alpha\mu/2\overline{E}$. It must also be that, with school A exerting zero effort, school B does not wish to jack up its effort level sufficient to start attracting students who can enroll in school A. This would require that school B exert an effort level considerably in excess of $\alpha\mu/2\overline{E} - \left(2\overline{E} - 1\right)\bar{c}$. Clearly, the larger is $\alpha\mu/2\overline{E} - \left(2\overline{E} - 1\right)\bar{c}$ the less likely will this deviation be attractive.

When neither the equilibrium of the baseline model or the zero effort equilibrium exist, then equilibrium will be in mixed strategies. Our procedure for solving for equilibrium in this case is outlined in the online Appendix. For the parameterization that underlies Figure 1 and a value of $\overline{E}$ of 0.7, we find that the equilibrium of the basic model survives for values of α in the interval $[0, 1.346]$ and the zero effort equilibrium exists for values α in the interval $[1.454, 2]$. A mixed strategy equilibrium exists for values of α in the interval $(1.346, 1.454)$. In this equilibrium, only school B randomizes. School A chooses a positive effort level, and school B randomizes between zero effort and a positive effort level. As α increases, the probability that school B chooses a positive effort converges to zero and the effort level chosen by school A converges to zero.

Figure 3 illustrates the impact of school choice in the model with capacity constraints. The four panels replicate those in Figure 1, although, given the uncertainty in school B's effort level when it is using a mixed strategy, we compute expected values of the outcome variables.[26] Again, the two figures look qualitatively quite similar and the lessons of the baseline model are basically robust to introducing capacity constraints. The only difference is that there is in fact no increase in average school quality or the quality of the school in the less affluent neighborhood, when peer preferences are strong enough that the capacity constraint binds.

[26]Note that the expected effort level of school B is equal to the effort level for school A in the mixed strategy equilibrium. It is straightforward to show that this must be true in this type of mixed strategy equilibrium.

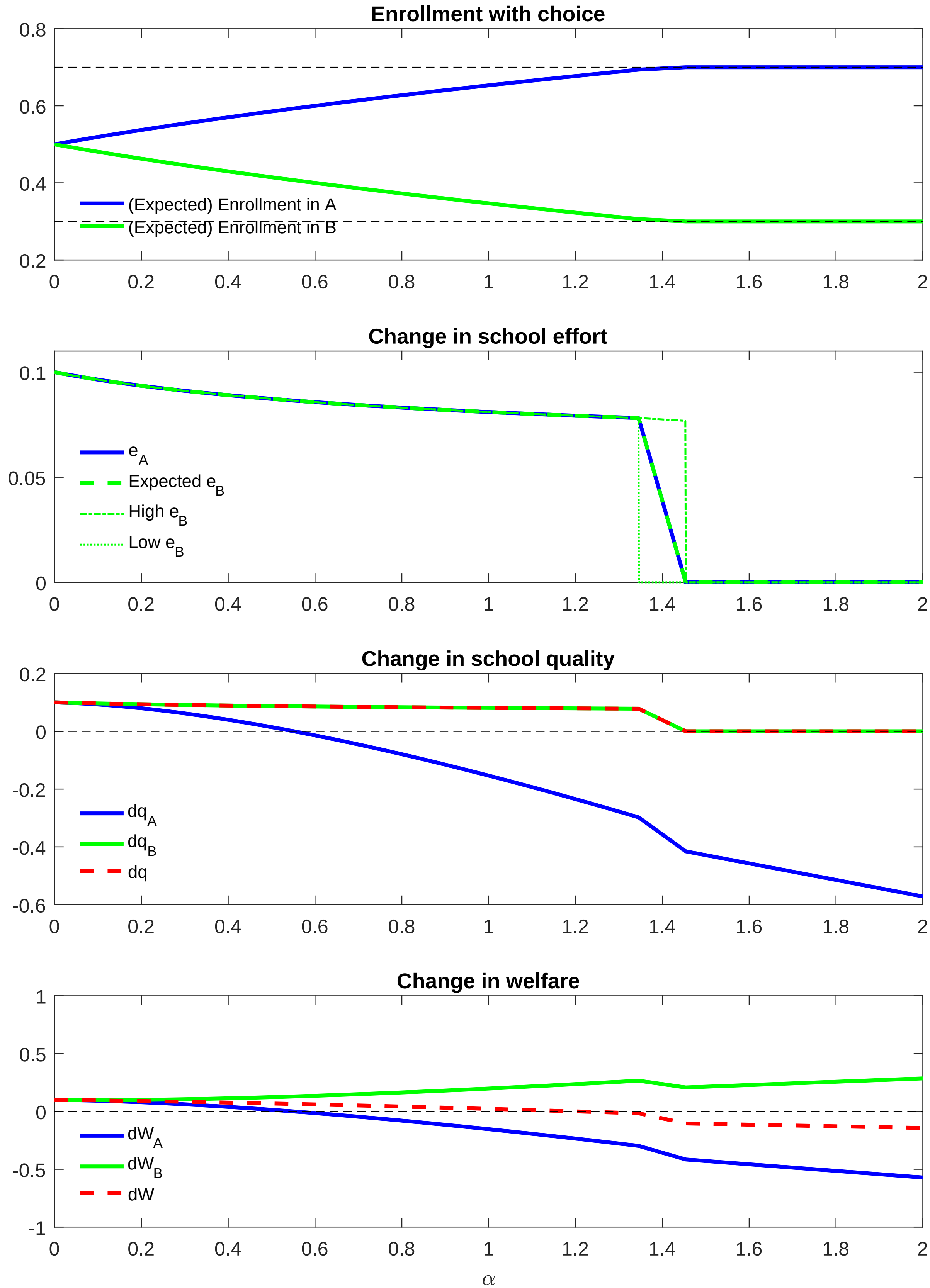

Figure 3: The impact of school choice in the model with capacity constraints

In this sense, it is fair to say that capacity constraints amplify the problems created for school choice by strong peer preferences and unequal neighborhoods.

6 Costs varying by socio-economic status

The baseline model assumes that the utility cost of exercising school choice is ex ante identical across socio-economic status. As noted in the introduction, it is often argued that more advantaged households are better able to take advantage of school choice. It is therefore desirable to extend the analysis to capture this. The simplest way of extending the model is to assume that a fraction of households are "immobile" and that immobility is higher among lower socio-economic status households. Intuitively, these immobile households are those who will never choose the non-neighborhood school because they have prohibitively high costs. Introducing cost heterogeneity in this way allows us to hold constant the enrollment response to any given quality differential, but vary the composition of the switching group.

Formally, we assume that in neighborhood B a fraction $\lambda - \theta\,(\mu/2 + s)$ of households of type s are immobile. The parameter λ is between 0 and 1 and the parameter θ is non-negative but less than the minimum of $2\lambda/\mu$ and $2(1-\lambda)/\mu$. When θ is equal to 0, the fraction of immobile households is constant across socio-economic status. When θ is positive, the fraction decreases as socio-economic status rises. The higher is θ, the greater is the correlation between low socio-economic status and immobility. Nonetheless, for all θ, the fraction of immobile households in neighborhood B is constant at λ.[27]

To see how this change impacts the analysis, suppose the anticipated quality differential Δq is non-negative. Then, households in neighborhood B will use school A if they are mobile and their costs are less than Δq. Accordingly, the fraction of households exercising choice is $(1-\lambda)\Delta q/2\overline{c}$ which is independent of θ. What depends on θ is the average socio-economic status of these households. Consider some type s residing in neighborhood B (i.e., belonging to the interval $[-\mu, 0]$). The probability that this type will exercise choice is $[1-\lambda+\theta(\mu/2+s)]\,\Delta q/\overline{c}$. The probability density of these types in the set of those exercising choice is therefore $[1-\lambda+\theta(\mu/2+s)]\,/(1-\lambda)$. With some work (see the on-line Appendix), it can be shown that this implies that the average type in the set of those exercising choice is

$$-\frac{\mu}{2} + \frac{\theta\mu^2}{12\,(1-\lambda)}. \tag{49}$$

Note that this is increasing in θ. This in turn implies that the average socio-economic status of school A's

[27] Since we are focusing on the switching of households from neighborhood B to school A under school choice, we simply assume that a constant fraction λ of neighborhood A's households are immobile.

students is

$$s_A = \frac{\mu}{2}\left[\frac{1-(1-\lambda)\frac{\Delta q}{\bar{c}} + \frac{\Delta q}{\bar{c}}\frac{\theta\mu}{6}}{1+(1-\lambda)\frac{\Delta q}{\bar{c}}}\right], \tag{50}$$

and that of school B's students is

$$s_B = -\frac{\mu}{2}\left[1 + \frac{\frac{\Delta q}{\bar{c}}\frac{\theta\mu}{6}}{1-(1-\lambda)\frac{\Delta q}{\bar{c}}}\right]. \tag{51}$$

Using (1), this means that, if households correctly anticipate other households' decisions, Δq must satisfy the equation

$$\Delta q = \Delta e + \frac{\alpha\mu}{1+(1-\lambda)\frac{\Delta q}{\bar{c}}} + \frac{\alpha\mu\frac{\Delta q}{\bar{c}}\frac{\theta\mu}{6}}{1-\left((1-\lambda)\frac{\Delta q}{\bar{c}}\right)^2}. \tag{52}$$

This gives rise to a cubic equation. It is possible that this equation has two positive solutions in the relevant range (i.e., satisfying $\Delta q < \bar{c}$). In such a situation, there are two possible equilibrium quality differentials. One involves a low quality differential and few students from the less affluent neighborhood exercising choice. The other involves a high quality differential and more students switching schools. This multiplicity is possible because the high socio-economic students are leaving the school in the less affluent neighborhood and hence lowering its quality when they leave. Because they are of lower socio-economic status than the students in the affluent neighborhood, they also lower the quality of school A. However, the reduction in the quality of school A can be smaller than the reduction in the quality of school B and this is what underlies the multiplicity. Nonetheless, this is only a possibility and does not arise in the numerical example analyzed below.

Given all this, with effort levels e_A and e_B, the two schools will anticipate enrollments of

$$E_A(\Delta e) = \frac{1}{2}\left[1+(1-\lambda)\frac{\Delta q(\Delta e)}{\bar{c}}\right], \tag{53}$$

and

$$E_B(\Delta e) = \frac{1}{2}\left[1-(1-\lambda)\frac{\Delta q(\Delta e)}{\bar{c}}\right]. \tag{54}$$

The equilibrium effort levels will be identical and given by

$$e_A^* = e_B^* = \frac{1}{\gamma}\left[\frac{(1-\lambda)}{2}\frac{\Delta q'(0)}{\bar{c}}\right]. \tag{55}$$

Computing the derivative from (52), we find that the equilibrium effort level is e_V^* (*effort under school choice* *with costs varying by socio-economic status*) which is defined to equal

$$e_V^* \equiv \frac{1}{\gamma}\left[\frac{1-\lambda}{2\left(\overline{c}+\frac{\alpha\mu(1-\lambda)}{\left(1+(1-\lambda)\frac{\Delta q(0)}{\overline{c}}\right)^2}-\frac{\alpha\mu\frac{\theta\mu}{6}\left(1+\left((1-\lambda)\frac{\Delta q(0)}{\overline{c}}\right)^2\right)}{\left(1-\left((1-\lambda)\frac{\Delta q(0)}{\overline{c}}\right)^2\right)^2}\right)}\right]. \tag{56}$$

Given (56), the equilibrium qualities of the two schools under school choice (q_A^*, q_B^*) will be given by

$$q_A^* = e_V^* + \frac{\alpha\mu}{2}\left[\frac{1-(1-\lambda)\frac{\Delta q(0)}{\overline{c}}+\frac{\Delta q(0)}{\overline{c}}\frac{\theta\mu}{6}}{1+(1-\lambda)\frac{\Delta q(0)}{\overline{c}}}\right], \tag{57}$$

and

$$q_B^* = e_V^* - \frac{\alpha\mu}{2}\left[1+\frac{\frac{\Delta q(0)}{\overline{c}}\frac{\theta\mu}{6}}{1-(1-\lambda)\frac{\Delta q(0)}{\overline{c}}}\right]. \tag{58}$$

Figure 4 is the counterpart to Figure 1. It assumes a value of λ equal to 0.5 and a value of θ equal to 0.95. These choices imply that a large fraction of households are immobile and that immobility is highly correlated with socio-economic status. Most of the key conclusions of the baseline model are robust to introducing costs that vary with socio-economic status. There are two differences to note. First, the change in average school quality is no longer everywhere decreasing in the strength of peer preferences. This is because, as illustrated by Panel B, the increase in school effort created by choice is actually increasing in α after some point. This reflects the fact that enrollment becomes more responsive to effort at higher levels of α. This in turn is a consequence of the fact that the socio-economic difference between the two schools becomes increasing in α.[28] Second, the quality of the disadvantaged school decreases when peer preferences are high.[29] This reflects the fact that the disadvantaged school is losing its higher socio-economic status students. This is precisely the concern expressed by critics of choice. Nonetheless, the welfare of households in the disadvantaged neighborhood increases. This increase is driven by the benefits enjoyed by those households exercising choice.

[28] In the baseline model, $\Delta q'(0)$ in equation (9) is decreasing in α. Intuitively, this reflects the fact that as α increases more households are exercising choice and the socio-economic difference between the two schools decreases. In the model of this Section, $\Delta q'(0)$ in equation (55) is eventually increasing in α. Intuitively, this reflects the fact that as more households exercise choice, the socio-economic difference between the two schools actually increases. This is because the disadvantaged school is losing its higher socio-economic status students.

[29] Average school quality increases despite the fact that quality in both schools goes down, because more students are at the higher quality school and average quality is enrollment weighted.

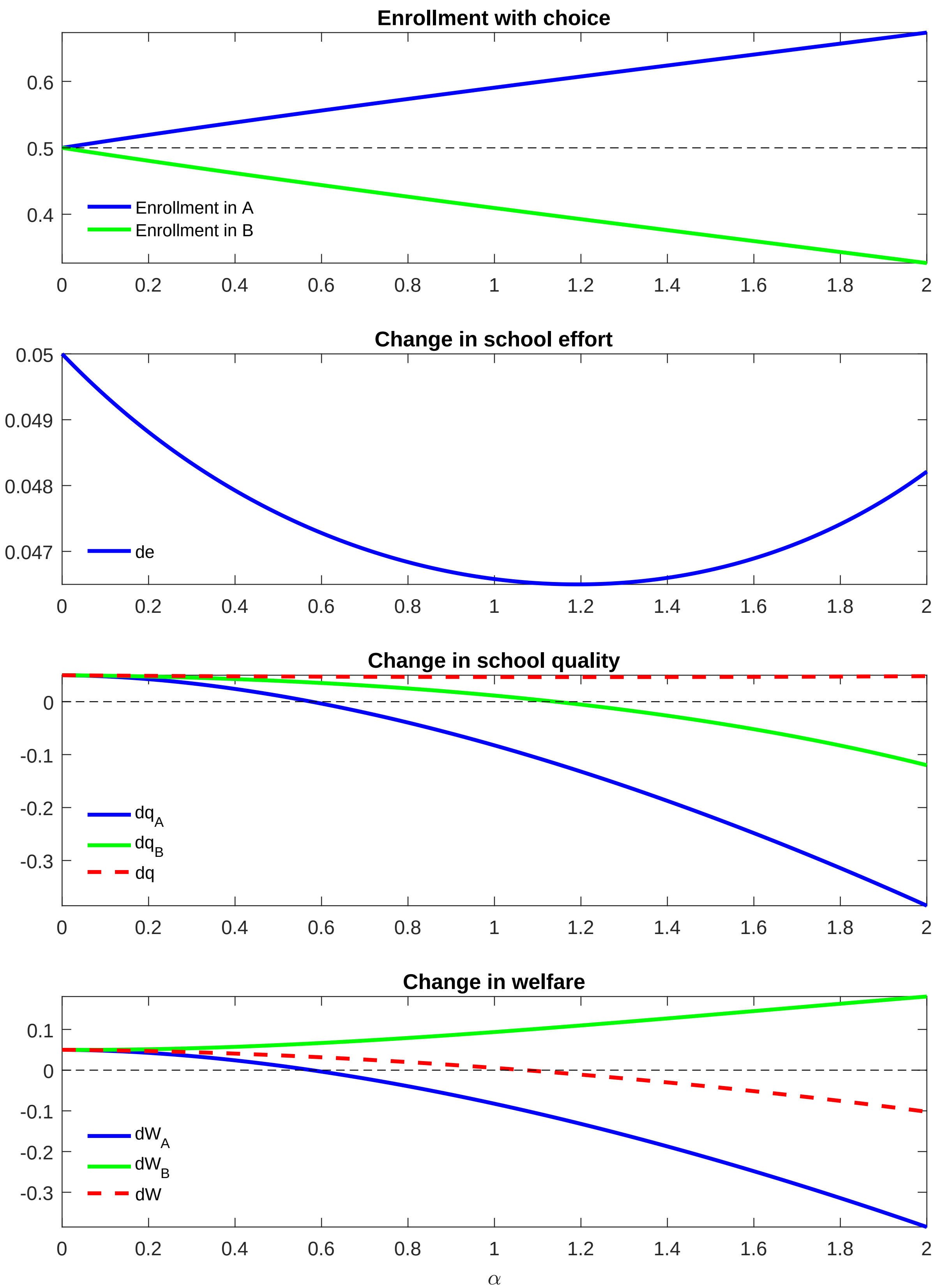

Figure 4: The impact of school choice in the model with costs varying by socio-economic status.

7 Conclusion

To recap, our baseline model generated three findings regarding the impact of public school choice. First, choice increases average school quality, but the increase is smaller when parents have stronger peer preferences and neighborhoods are more unequal. Average quality increases because the competition for enrollment unleashed by choice spurs schools to increase effort. Equilibrium effort levels are higher, the greater the responsiveness of enrollment to effort. When peer preferences are stronger and neighborhoods less equal, enrollment decisions are more driven by peer group concerns than differences in school efforts. Enrollment is therefore less responsive to effort, resulting in lower equilibrium effort levels.

Second, public school choice can increase or decrease aggregate welfare. It increases welfare when households have weak peer preferences or neighborhood inequality is small and decreases welfare when parents have strong preferences and neighborhoods are unequal. This reflects that while the threat of choice is socially beneficial (since it elicits socially beneficial effort), the exercise of choice is not (since it costly and since peer quality is zero-sum construct).[30] Peer preferences weaken the threat posed by choice (because enrollment decisions are more driven by peer group concerns), but also increase the likelihood that choice is exercised. That is because, in equilibrium, the two schools exert identical levels of effort, meaning that differences in school quality - and hence the extent to which choice is exercised - are driven by peer preferences and neighborhood inequality.

Third, public school choice always benefits the less affluent neighborhood. The quality of its school increases and the welfare of its households increases. By contrast, when peer preferences are stronger and neighborhoods are less equal, choice harms the more affluent neighborhood. The quality of its school decreases and this causes the welfare of its households to decrease. This decrease in quality reflects the fact that the benefits of greater effort by school personnel are offset by the costs of a worse peer group.

Our baseline model ignored the incentives for schools created by neighborhood choice. It also abstracted from school capacity constraints and assumed that the costs of exercising choice were independent of socio-economic status. Nonetheless, our subsequent analysis showed that, roughly speaking, these three findings remain in our more complex models that incorporate these features. There are only three caveats. First, with capacity constraints, when peer preferences are strong and neighborhoods unequal, choice will not increase school quality. This is because the search for better peers, will leave the school in the more affluent neighborhood at capacity even when it exerts no effort. This destroys incentives for both schools. Second, when the costs of exercising choice are lower for higher socio-economic status households, choice may reduce

[30] Put differently, we would like to subject schools to the threat of choice but would like to prevent households from actually exercising choice. The two goals are obviously incompatible.

the quality of the school in the less affluent neighborhood. This is because the costs of losing its high socio-economic students may exceed the benefits from increased efforts from school personnel.[31] Third, again when the costs of exercising choice are lower for higher socio-economic status households, the change in average school quality is not everywhere decreasing in the strength of peer preferences. At some point, higher peer preferences raise the enrollment response to effort. The robustness of our findings lead us to view the forces revealed by our baseline model to be fairly general.

From a policy perspective, our analysis provides strong support for the position that public school choice is desirable in settings where peer preferences are weak and/or neighborhood inequality is low. It also provides support for the position that strong peer preferences combined with neighborhood inequality complicates the case for choice. Nonetheless, since the analysis suggests that choice always benefits households in disadvantaged neighborhoods, it may still be justified even in these settings. If policy-makers wish to benefit households in disadvantaged neighborhoods, the appropriate question is whether other policies are available that can create these benefits at lower cost to households in the advantaged neighborhoods (Coate (2000)).

The justification for choice as policy to benefit households in disadvantaged neighborhoods would be stronger if, absent choice, schools in the disadvantaged neighborhood were even less desirable. In our framework, schools in the disadvantaged neighborhood are of lower quality because they enroll more disadvantaged students. In practice, they may also exert less effort. This could be because disadvantaged households exert less "voice" pressure.[32] Alternatively, if effort is interpreted more broadly, it could be because their weaker composition makes it harder for them to attract good principals and teachers. In either case, choice would generate larger (reallocation-driven) welfare improvements among households in the disadvantaged neighborhood.

An obvious question is whether public school choice can be improved by auxiliary policies that affect how it operates. One option is to help disadvantaged students exercise choice (e.g., by subsidizing the costs of low socio-economic status students). The assumption underlying this policy is that, as in the model of Section 6, more advantaged students face a lower cost of using choice. Such a policy will increase the enrollment response to school efforts and thus should increase the equilibrium effort level.[33] However, it is not clear how

[31] Again, we stress that, the first and second caveats not withstanding, choice increases the welfare of households in the less affluent neighborhood. This welfare gain reflects the benefits obtained by those households exercising choice.

[32] McMillan (2005) assumes that households exert voice pressure, and that less-advantaged households exert less voice pressure. Ferreyra and Liang (2012) make a similar assumption. As pointed out by a referee, households that attend neighborhood schools might face lower costs and higher returns to voice investments (e.g., because they face lower costs of traveling to PTA meetings and because higher school quality increases neighborhood house prices). As such, they would be expected to invest more. Because stronger peer preferences lead to more households to attend the non-neighborhood school, this is another mechanism through which peer preferences could reduce the incentive effects of choice.

[33] There are two important caveats here. First, this assumes that the enrollment response does not trigger capacity constraints. If this happens, the policy could lower school effort. Second, if the increased enrollment response of low socio-economic status students comes at the expense of a reduced enrollment response from high socio-economic status students (say, because of a redistribution of expenditures aimed at publicizing the choice option), school effort will be lower. This can be shown formally

it will impact welfare when peer preferences are strong because it will induce more costly switching. Also the costs of such a policy would have to be included in a proper welfare analysis.[34] Another option is to provide schools with incentives to enroll lower socio-economic status students. Many school financing formula already have this feature, under the assumption that it costs more to educate certain types of students (e.g., those with special educational needs). Here the goal would be to increase the incentive to schools to enroll those students who are on the margin of exercising choice. Intuitively, a well-designed policy of this form seems likely to raise school effort and thus may be helpful.[35]

As we have stressed throughout, our analysis pertains to only one type of policy - public school choice. We studied this because it is the most widely-used choice policy. However, many would argue that it does not go far enough. For example, one could argue that school choice policies would be more effective if schools could choose which curricula to follow, if they had more flexibility over staffing decisions, or if they could take other actions to shape the size and composition of the student body. While this type of analysis is beyond the scope of this paper, we think our findings demonstrate that any such analysis would have to grapple with the complications created by peer preferences.

by reducing the θ parameter in the model of Section 6.

[34]Some feel for the impact of such a policy can be obtained by comparing Figures 1 and 4. Figure 1 assumes that the costs of exercising choice are independent of socio-economic status, while Figure 4 assumes that some households are immobile and that immobility is inversely correlated with socio-economic status. Thus, moving from Figure 4 to Figure 1 effectively involves lowering the costs of exercising choice, particularly for lower socio-economic status households. It is clear that school effort is higher in Figure 1. It is unclear what will happen to aggregate welfare, even ignoring the costs of such a policy.

[35]While space constraints prevent it, such a policy could be analyzed with our model. Our baseline model assumes that each school receives a per-student surplus of one. We could change that by, say, assuming that for a student of socio-economic status s a school receives a per-student surplus of $1 - s/\mu$. Thus, lower socio-economic status students yield higher surplus. Such a policy would keep the community's total payments to schools the same and hence be budget neutral. It seems clear that this policy will provide greater incentive for school effort. What is not clear is what is the optimal payment schedule and whether it is possible to reverse the conclusions about choice decreasing welfare when peer preferences are strong.

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A Appendix

A.1 Proof of Proposition 1

For parts (i) and (iii), we have from (15), (16), and (10), that

$$dq_B = dq = e_S^* = \frac{1}{4\gamma}\left[\frac{1}{\sqrt{\bar{c}^2 + 4\bar{c}\alpha\mu}} + \frac{1}{\bar{c}}\right] > 0.$$

It follows that

$$\frac{dq_B}{d\alpha\mu} = \frac{dq}{d\alpha\mu} = -\frac{\bar{c}}{2\gamma\left(\bar{c}^2 + 4\bar{c}\alpha\mu\right)^{\frac{3}{2}}} < 0.$$

For part (ii), we have from (14), that

$$dq_A = e_S^* - \alpha\mu\left(\frac{\frac{\Delta q(0)}{\bar{c}}}{1 + \frac{\Delta q(0)}{\bar{c}}}\right).$$

Note from (6) that

$$\frac{\frac{\Delta q(0)}{\bar{c}}}{1 + \frac{\Delta q(0)}{\bar{c}}} = \frac{\sqrt{\bar{c}^2 + 4\alpha\mu\bar{c}} - \bar{c}}{\sqrt{\bar{c}^2 + 4\alpha\mu\bar{c}} + \bar{c}}.$$

Thus, using (10), and writing the quality change in school A as a function of $\alpha\mu$, we have that

$$dq_A\left(\alpha\mu\right) = \frac{1}{4\gamma}\left[\frac{1}{\sqrt{\bar{c}^2 + 4\bar{c}\alpha\mu}} + \frac{1}{\bar{c}}\right] - \alpha\mu\left(\frac{\sqrt{\bar{c}^2 + 4\alpha\mu\bar{c}} - \bar{c}}{\sqrt{\bar{c}^2 + 4\alpha\mu\bar{c}} + \bar{c}}\right).$$

Note that

$$dq_A\left(0\right) = \frac{1}{2\gamma\bar{c}} > 0$$

and that

$$dq_A\left(\frac{3}{4}\bar{c}\right) = \frac{3}{8\gamma\bar{c}} - \frac{\bar{c}}{4} < 0,$$

where the last inequality follows from the assumption that $\bar{c} > \sqrt{6/\gamma}$. In addition, it is the case that

$$\frac{dq_A\left(\alpha\mu\right)}{d\alpha\mu} = -\frac{\bar{c}}{2\gamma\left(\bar{c}^2 + 4\bar{c}\alpha\mu\right)^{\frac{3}{2}}} - \left(\frac{\sqrt{\bar{c}^2 + 4\alpha\mu\bar{c}} - \bar{c}}{\sqrt{\bar{c}^2 + 4\alpha\mu\bar{c}} + \bar{c}}\right) - \alpha\mu\left(\frac{4\bar{c}^2\left(\bar{c}^2 + 4\alpha\mu\bar{c}\right)^{-\frac{3}{2}}}{\left(\sqrt{\bar{c}^2 + 4\alpha\mu\bar{c}} + \bar{c}\right)^2}\right) < 0,$$

so that $dq_A\left(\alpha\mu\right)$ is decreasing. It follows that there exists a unique value of $\alpha\mu$ less than $\frac{3}{4}\bar{c}$, say $\widetilde{\alpha\mu}$, such that $dq_A\left(\alpha\mu\right)$ is positive for $\alpha\mu \in [0, \widetilde{\alpha\mu})$ and negative for $\alpha\mu \in (\widetilde{\alpha\mu}, \bar{c}]$. ■

A.2 Proof of Proposition 2

For part (i), using (19), (16), and (7), we have that

$$dW = e_S^* - \frac{\Delta q(0)^2}{4\bar{c}}.$$

From (6), we have that

$$\frac{\Delta q(0)^2}{4\bar{c}} = \frac{\bar{c} + 2\alpha\mu - \sqrt{\bar{c}^2 + 4\alpha\mu\bar{c}}}{8}.$$

Thus, using (10), we can write the change in average welfare as a function of $\alpha\mu$ as

$$dW(\alpha\mu) = \frac{1}{4\gamma}\left[\frac{1}{\sqrt{\bar{c}^2 + 4\bar{c}\alpha\mu}} + \frac{1}{\bar{c}}\right] - \left(\frac{\bar{c} + 2\alpha\mu - \sqrt{\bar{c}^2 + 4\alpha\mu\bar{c}}}{8}\right).$$

Note that

$$dW(0) = \frac{1}{2\gamma\bar{c}} > 0,$$

and that

$$dW(\frac{3}{4}\bar{c}) = \frac{3}{8\gamma\bar{c}} - \frac{\bar{c}}{16} < 0$$

where the last inequality follows from the assumption that $\bar{c} > \sqrt{6/\gamma}$. In addition, we have that

$$\frac{dW(\alpha\mu)}{d\alpha\mu} = -\frac{\bar{c}}{2\gamma\left(\bar{c}^2 + 4\bar{c}\alpha\mu\right)^{\frac{3}{2}}} - \frac{1}{4} + \frac{\bar{c}}{4\sqrt{\bar{c}^2 + 4\alpha\mu\bar{c}}} < 0,$$

so that $dW(\alpha\mu)$ is decreasing. It follows that there exists a unique value of $\alpha\mu$ less than $\frac{3}{4}\bar{c}$, say $\widetilde{\alpha\mu}$, such that $dW(\alpha\mu)$ is positive for $\alpha\mu \in [0, \widetilde{\alpha\mu})$ and negative for $\alpha\mu \in (\widetilde{\alpha\mu}, \bar{c}]$.

For part (ii), we know from (17) that

$$dW_A = dq_A,$$

so the result follows from Proposition 1.

For part (iii), we know from (18) that

$$dW_B = \left(1 - \frac{\Delta q(0)}{\bar{c}}\right) dq_B + \int_0^{\Delta q(0)} (dq_A + \alpha\mu - c)\,\frac{dc}{\bar{c}}.$$

From (15), we know that dq_B is positive. In addition, for all $c \in [0, \Delta q(0)]$ it is the case that $dq_A + \alpha\mu - c$ is at least as big as dq_B. Thus, dW_B is positive. In addition, from (14) and (15), we have that

$$dq_A = e_S^* - \alpha\mu\left(\frac{\frac{\Delta q(0)}{\overline{c}}}{1+\frac{\Delta q(0)}{\overline{c}}}\right),$$

and that

$$dq_B = e_S^*.$$

Using these, we can write

$$dW_B = e_S^* + \alpha\mu\left(\frac{\frac{\Delta q(0)}{\overline{c}}}{1+\frac{\Delta q(0)}{\overline{c}}}\right) - \frac{\Delta q(0)^2}{2\overline{c}}.$$

Using (6) and (10), we can write dW_B as a function of $\alpha\mu$ as follows:

$$dW_B(\alpha\mu) = \frac{1}{4\gamma}\left[\frac{1}{\sqrt{\overline{c}^2+4\overline{c}\alpha\mu}}+\frac{1}{\overline{c}}\right] + \alpha\mu\left[\frac{\sqrt{\overline{c}^2+4\alpha\mu\overline{c}}-\overline{c}}{\sqrt{\overline{c}^2+4\alpha\mu\overline{c}}+\overline{c}}\right] - \left[\frac{\overline{c}+2\alpha\mu-\sqrt{\overline{c}^2+4\alpha\mu\overline{c}}}{4}\right].$$

Note that

$$dW_B(0) = \frac{1}{2\gamma\overline{c}} > 0,$$

and that

$$dW_B\left(\frac{3}{4}\overline{c}\right) = \frac{3}{8\gamma\overline{c}} + \frac{\overline{c}}{8}.$$

Moreover, we have that

$$dW_B(0) - dW_B\left(\frac{3}{4}\overline{c}\right) = \frac{1}{8\gamma\overline{c}} - \frac{\overline{c}}{8} < 0$$

where the latter inequality follows from the assumption that $\overline{c} > \sqrt{6/\gamma}$. Differentiating, we obtain

$$\frac{dW_B(\alpha\mu)}{d\alpha\mu} = -\frac{\overline{c}}{2\gamma\left(\overline{c}^2+4\overline{c}\alpha\mu\right)^{\frac{3}{2}}} + \left[\frac{\sqrt{\overline{c}^2+4\alpha\mu\overline{c}}-\overline{c}}{\sqrt{\overline{c}^2+4\alpha\mu\overline{c}}+\overline{c}}\right] + \alpha\mu\left(\frac{4\overline{c}^2\left(\overline{c}^2+4\alpha\mu\overline{c}\right)^{-\frac{3}{2}}}{\left(\sqrt{\overline{c}^2+4\alpha\mu\overline{c}}+\overline{c}\right)^2}\right) - \frac{1}{2} + \left(\frac{\overline{c}}{2\sqrt{\overline{c}^2+4\alpha\mu\overline{c}}}\right).$$

This derivative is difficult to sign, since it consists of both positive and negative terms. However, note that

$$\frac{dW_B(0)}{d\alpha\mu} = -\frac{1}{2\gamma\overline{c}^2} < 0.$$

It follows that dW_B is decreasing in $\alpha\mu$ for sufficiently small $\alpha\mu$ but then, since $dW_B(0) < dW_B\left(\frac{3}{4}\overline{c}\right)$, must at some point must be increasing in $\alpha\mu$. ■