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Community Development by Public Wealth Accumulation*

Abstract

This paper presents a dynamic political economy model of community development. The model highlights the role of public wealth accumulation in development. A community's wealth is the difference between the value of its publicly-owned assets and liabilities. Wealth accumulation arises when residents tax-finance investment in durable public goods or pay down public debt. It stimulates future development because of the fiscal externality created by the collective ownership of community wealth. Moreover, when this development occurs, future residents have an incentive to engage in further accumulation because the cost can be spread over a larger group. In this way, public wealth accumulation can fuel gradual community development.

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1 Introduction

Community development is an important topic in public and urban economics. In order for development to occur, it is necessary for housing to be built for new residents to live in and investments to be made in local public goods (schools, roads, public safety, parks, libraries, shopping and entertainment areas, etc) for their use. How this occurs and the distortions that arise in different institutional settings is a question of significant interest.

This paper presents and analyzes a novel political economy model of community development. It is a dynamic partial equilibrium model of a single community. The community starts out with a stock of housing and initial residents who own this housing. The community can grow by building new housing and new construction is supplied by competitive developers. There is a pool of potential residents with heterogeneous desires to live in the community, generating a downward sloping demand curve. There is turnover, with households entering and exiting the pool each period, so that the market for housing is always active. The possibility that current residents may leave the pool gives them an incentive to care about the value of their homes. The community provides a durable local public good, investment in which can be financed by debt or a uniform tax on all residents. Public investment and financing decisions are made in each period by the current residents and residents are fully forward-looking.

The main insight from the analysis concerns the potential role of public wealth accumulation in community development. The model's structure implies that the community has public assets and liabilities in the form of a stock of public good and public debt. Ownership of these assets and liabilities is collective by the residents of the community. This collective ownership creates what is known as a *fiscal externality* (see, for example, Henderson 1988 p.166). Specifically, building an additional house in the community creates effectively a transfer (positive or negative) from the new resident to the existing residents. If the community's wealth (i.e., the value of its public good stock less its debt) is positive, adding a new resident will reduce wealth per capita and the fiscal externality is negative. If wealth is negative, adding a new resident will increase wealth per capita and the fiscal externality will be positive. This externality is analogous to that arising in a common pool problem (although it can be positive) and will potentially distort the decisions of potential residents to build in the community.

Of course, the community's wealth is endogenous and depends on its past fiscal decisions. These decisions shape the sign and strength of the externality and make it conceptually distinct from other fiscal externalities or common pool problems. When making fiscal decisions, forward-looking residents take into account their future impact on the externality. If the community has negative wealth, residents may want to increase it to benefit from the positive externality. For example, if they can attract more residents by reducing the community's debt partially, this will spread the burden of the remaining debt over a larger group of residents. More importantly, even when the community has positive wealth, residents may want to further increase it to attract potential residents to benefit from a second fiscal externality - that created by new residents sharing the costs of public good provision.

To improve the community's wealth requires residents to either tax-finance investment in the public good or raise taxes to pay down debt. In doing so, they are effectively subsidizing future residents. They will not always want to engage in this behavior, but, if they do, they create development for the community beyond the level that could be supported with its initial wealth. Moreover, if such development occurs, future residents may have an incentive to engage in more accumulation. This is because the cost of further accumulation can be spread over a larger group of residents. In this way, public wealth accumulation can fuel the gradual development of the community.

The paper's formal analysis begins by finding an equilibrium of the model. Following the usual approach in dynamic political economy models, we look for a Markov equilibrium in which residents' policy decisions and housing market outcomes just depend on the state variables - the public assets and housing stock. The novelty of the model makes finding an equilibrium a significant challenge. Our strategy is "guess and verify". To develop intuition for what to guess, we characterize how development would proceed if the initial residents of the community could commit future residents to following a complete development plan. This plan is relatively simple and reveals the economics underlying the strategy of public wealth accumulation in its simplest form. We then consider the time consistency of this plan to see how it needs to be amended in the presence of sequential decision-making. We find that if the community's initial wealth is below a critical level, future residents will not want to follow the initial residents' optimal plan. They will desire additional public wealth accumulation because its cost is lower with a larger population. On the basis of this understanding of the initial residents' plan and its time inconsistency, we formulate a guess for an equilibrium and verify analytically that this is indeed an equilibrium. We then discuss existence, verifying by numerical methods that our equilibrium exists for a broad range of parameterizations of our model.

Once we have this equilibrium, we describe how the community develops for any initial conditions (i.e., initial public assets and housing stock). We then clarify the circumstances under which equilibrium involves community development by public wealth accumulation. This is defined as arising whenever the community grows beyond the size that could be supported by its initial wealth. We show that this occurs if and only if the community's initial wealth falls below a critical level. We further show that this development comes in one of two forms. In the first, development begins in the initial period and the size of the community increases gradually and continually over time, converging asymptotically to a steady state level. In each period, the residents build the community's wealth by financing some of the additional public good provided for the larger population with taxation. This accumulation paves the way for more development in the next period. The logic behind this gradual development is subtle because it is the off-equilibrium path behavior of future residents that provides incentives to current residents to engage in wealth accumulation. In the second form, there is no development in the first period and taxes are raised to build wealth. This accumulation allows development to begin in the next period. This development can either be gradual or rapid depending on the extent of accumulation in the initial period. Either way, the community converges to the same steady state.

Finally, we turn to the normative potential of community development by public wealth accumulation. We show that it does not allow the community to reach its efficient size. This reflects the fact that the future benefits of accumulation necessarily accrue partially to future residents via the fiscal externality. Thus current residents do not fully appropriate these benefits. These forces mean that accumulation will be under-supplied by residents. Nonetheless, the difference between the long run size of the community and the efficient size converges to zero as the probability of residents leaving the pool of potential residents converges to zero. We also show that, irrespective of the eventual size of the community, development by public wealth accumulation involves inefficient delay.

The organization of the remainder of the paper is as follows. Section 2 discusses related literature. Section 3 introduces the model and Section 4 establishes a normative benchmark by characterizing socially optimal development. Section 5 explains our strategy for finding equilibrium and describes the equilibrium that we uncover. Section 6 analyzes the conditions under which this equilibrium involves community development by public wealth accumulation and the nature of this development. Section 7 investigates how such development compares with optimal development and the reasons for the discrepancies. Section 8 discusses the empirical plausibility of development by wealth accumulation and identifies some other lessons from the analysis. Section 9 concludes with a brief summary and suggestions for future work.

2 Related literature

This paper relates to a variety of literatures in public economics, urban economics, and political economy. A long tradition in public economics sees the role of communities as facilitating the joint consumption of local public goods. Accordingly, a large literature studies the formation and development of communities which independently finance and provide their local public goods (see Ross and Yinger 1999 and Wildasin 1986 for reviews). The most prominent strand of this literature analyzes how households who differ in incomes or public good preferences or both, locate across different communities under the assumption that, once located, households collectively choose taxes and public good levels for their communities and optimize their housing consumption (see, for example, Epple, Filimon, and Romer 1984, Fernandez and Rogerson 1998, and Rose-Ackerman 1979). The number of communities and the housing supplies in each community are exogenous and the focus is on the existence of equilibrium and on whether the population is segmented across communities in the way envisioned by Tiebout (1956). Our model shares with this literature the view that the role of communities is to facilitate the joint consumption of local public goods. It also has in common the assumption that fiscal decisions are made by residents. It differs in that its focus is on the dynamic development of a single community rather than the allocation of households across multiple communities.

Another strand of this literature takes a different approach to the community formation process by assuming that communities are formed by monopoly developers (see, for example, Henderson 1985 and Sonstelie and Portney 1978). These developers acquire land, build housing, and provide public goods with the aim of attracting residents and making profits. They face a pool of potential residents who must be provided with at least some target utility level to be induced to locate in their communities. Developers' profits are the revenue from selling property less the costs of construction and public good provision. This literature studies the efficiency of public good provision, housing levels, and the allocation of households across communities. Our model shares with this literature the idea that community decisions are made strategically with an eye on how they will attract potential residents. It differs in the sense that decisions are made by residents, rather than monopoly developers. Moreover, these residents have only imperfect control over the level of new construction, which is supplied by competitive developers.

While there is recognition of its importance, the dynamic development of communities has received limited attention in this literature. Henderson (1980) studies the developer's problem in a two period, single community setting. There are two groups of potential residents: those around in the first period and those arriving in the second. The developer sells homes in the initial period to the period one potential residents, sells further homes in period two to the second group, and provides public goods in both periods. Period one residents remain in the community for both periods and public good provision is financed by a property tax in each period. Henderson shows that if the developer can commit to policies at the beginning of period one, the home sizes and public good levels provided in both periods will be efficient.[1] In our model, the residents are the decision-makers and decision-making is sequential, so there is no commitment. Moreover, homes come in one variety and, while the set of potential residents is changing, its size is constant across periods. Finally, and most significantly, because the public good is durable and the community can borrow, the community has assets and liabilities. These create the fiscal externality that drives development.

The urban economics literature explores reasons for agglomeration other than the collective consumption of local public goods. The famous monocentric city model originating in the works of Alonso (1964), Mills (1967), and Muth (1969), assumes that households agglomerate to be near their place of work in the city center. In the influential work of Krugman (1991), producers of differentiated products and worker/consumers with tastes for variety agglomerate to be able to produce, work, and trade with lower transactions costs. Some work in this literature develops dynamic models of city development. For example, a number of authors have studied development in the monocentric city model under the assumption that population is growing exogenously (for a review of this work, see Brueckner 2000). Of particular interest is the work of Henderson and Venables (2009) who consider a model in which new population arrives continuously and cities form sequentially. Each city has the structure of the monocentric model, but the model allows for other forms of agglomeration economies. As in our model, housing, once constructed, is infinitely durable. The planning solution involves one city after another being created and filled to optimal size with arriving citizens. A decentralized equilibrium is studied in which construction, as in our

[1] Henderson notes a tension between the interests of the developer and the period one residents. At the beginning of period two, the developer would like to shift more of the second period tax burden to initial residents and hence extract higher sales revenues from period two residents. He points out that this tension creates a potential time inconsistency problem. Our model also displays a time inconsistency problem, in the sense that current residents prefer that future residents choose different policies than they do.

model, is chosen by competitive builders and there are no local city governments to coordinate development. In this equilibrium, cities could be too small or too large. The paper then studies an equilibrium in which profit maximizing city developers offer subsidies to residents to live in their cities. Developers commit to these subsidies before their cities are formed and subsidies vary over the course of city development. This equilibrium replicates the planning solution.

This paper differs from this literature in its focus on the development of a small community in which people live, rather than a city where people also produce and trade. More importantly, what is distinctive about this paper is its modelling of community policy-making (specifically, sequential decision-making by home-owning residents) and its incorporation of community assets and liabilities (a durable public good and debt). This set-up leads to community development by public wealth accumulation. This completely novel type of development emerges organically rather than being driven by exogenous changes in the environment, such as population growth.

Turning back to public economics, the paper relates to the literature on the capitalization of local government amenities (school quality, taxes, crime, etc) into housing prices stemming from Oates (1969). The structure of the model implies that such capitalization is operational, but incomplete. In any period, the supply curve of housing looks like an inverted L. It is vertical up to a price equal to the construction cost with a quantity equal to the current housing stock, and then becomes horizontal, as new construction is added. This implies that an improvement in the community's amenities will not be capitalized into housing prices if demand prior to the improvement is already sufficient to bring forth new construction. The increase in demand will just be met by an increase in new construction. However, if even after the improvement, demand is insufficient to spur new construction, then capitalization will be just as in a model with fixed housing supply. This incomplete capitalization has interesting implications for residents' incentives to invest and borrow.

Another related public economics literature, is that on the political economy of public good provision. This literature explores how public good provision is determined in different political settings (see, for example, Baron 1996, Bergstrom 1979, Lizzeri and Persico 2001, and Romer and Rosenthal 1979). Most of the work, including the state and local literature just discussed, focuses on the provision of static public goods, which must be provided anew each period. In practice, many important public goods are durable, lasting for many years and depreciating relatively slowly. Understanding the political provision of such goods is more challenging, because of their durable nature. Recent work has studied the provision of such goods in a variety of settings.[2] This paper adds to this strand of the literature by studying the provision of a local durable public good by residents in a growing community with population turnover. In the equilibrium we find, despite population turnover and incomplete capitalization, investment is always efficient given the size of the community.[3] This reflects the assumption that the government can finance investment with debt.

Also related, is the literature on the political economy of public debt. A large literature studies the accumulation of debt at the national government level in various political settings (for a review see Alesina and Passalacqua 2016). The key focus has been on understanding why debt accumulation may be excessive. Less attention has been paid to the debt of local governments. A notable feature of such debt is that, once a resident leaves the locality, he/she ceases to have any responsibility for it. One theme in the literature is that this may result in the costs of debt not being fully bourne by the issuing residents. On the other hand, in a world in which the supply of housing is fixed, local government debt should be capitalized into the price of housing, putting the full burden of debt on the issuing residents even if they leave (see, for example, Daly 1969). As just discussed, in the model of this paper, capitalization is incomplete. Nonetheless, the fact that higher debt levels are capitalized into housing prices if new construction is not undertaken, prevents debt from being abused. In particular, residents do not increase debt and use it to finance tax cuts for themselves because this would deter development and lead to a fall in the value of their homes. In equilibrium, the wealth of the community either stays constant or grows over time and debt plays a key role in both allowing the community to develop and to provide public goods efficiently.[4]

[2] Battaglini and Coate (2007), Battaglini, Nunnari, and Palfrey (2012), and LeBlanc, Snyder, and Tripathi (2000) study legislative provision, Barseghyan and Coate (2014) and Coate and Ma (2017) study bureaucratic provision, and Conley, Driskill, and Wang (2013) and Schultz and Sjostrom (2001) study provision in a multi-community setting.

[3] In a community with a fixed housing supply and population turnover, capitalization provides incentives for residents to make efficient public good investment decisions despite the fact that they will not be around to enjoy all the benefits. See Brueckner and Joo (1991) and Conley, Driskill, and Wang (2013) for further discussion of the incentives provided by capitalization for public good provision.

[4] The role played by debt in the model differs from the role commonly discussed in the literature. The usual argument is that because residents may leave their community, they will underinvest in durable public goods if they must finance investment with taxes. Debt financing counteracts this distortion because mobility also implies that residents do not bear the full burden of debt issued (for a formal analysis see Schultz and Sjostrom 2001). This logic underlies the so-called "golden rule" that prescribes that local governments pay for non-durable goods and services with tax revenues and use debt to finance investment in durables (for discussion and analysis see Bassetto with Sargent 2006). In our model, no golden rule is imposed and residents finance investment with a mix of debt and taxes. Debt allows the community to accumulate wealth in a way that does not require distorting public good investment. Wealth accumulation is necessary to attract new residents.

A final related public economics literature is that on clubs, particularly club dynamics. Roberts (2015) considers a club that provides a public good and shares the costs of provision among its members. Potential members differ in the strength of their public good preferences. Each period, the existing members (who are those with the strongest public good preferences), must decide how many new members to admit. All potential members would like to be admitted, and, once admitted, remain in the club indefinitely. After new members have been admitted, the membership decides on a level of public good to provide. The public good is non-durable and club decisions are made via majority rule. The existing members face a trade-off: if they admit more members, this reduces the per-member cost of the public good, but it also changes the provision level because new members have weaker preferences. This trade-off gives rise to interesting equilibrium membership dynamics. While there are many differences, the model of this paper shares with Robert's work that the key rationale for the community is the sharing of the costs of a public good, and, that the dynamic structure of community decision-making involves existing residents choosing policies which determine next period's residents.[5]

Finally, the paper contributes to the fast growing literature developing and analyzing infinite horizon political economy models of policy-making with rational, forward-looking decision makers.[6] It is well recognized that many interesting issues arise from recognizing the dynamic linkage of policies across periods. Such linkages arise directly, as with public investment or debt, or indirectly because current policy choices impact citizens' private investment decisions. The model studied in this paper is distinctive in featuring both state variables directly controlled by the voters (debt and the stock of public good) and a state variable determined by the market (the housing stock). It also features a changing group of decision-makers, as the size of the community is growing. Despite these complications, we are able to provide something very close to a closed form solution of the model.

[5] The differences are first, that there is no conflict among residents concerning policy: all residents, irrespective of how long they have lived in the community, have the same policy preferences. Second, that new residents are free to move to the community if they are willing to buy a house, so residents control membership of the community only indirectly via the impact of their policy choices on the housing market. Third, that existing residents may need to leave the community, in which case they sell their houses. Fourth, that the public good is durable and the cost of investment can be shifted intertemporally via debt. Fifth, that the problem faced by the existing residents is that they need to attract new residents and this requires offering a higher level of public good surplus.

[6] Examples of this style of work are Azzimonti (2011), Battaglini and Coate (2008), Bowen, Chen, and Eraslan (2015), Coate and Morris (1999), Hassler, Rodriguez Mora, Storesletten, and Zilibotti (2003), and Krusell and Rios-Rull (1999). Examples which share the focus of this paper on state and local public finance are Barseghyan and Coate (2016) and Brinkman, Coen-Pirani, and Sieg (2016).

3 The model

Consider a community such as a small town or village.[7] This community can be thought of as one of a number in a particular geographic area. The time horizon is infinite and periods are indexed by $t = 0, ..., \infty$. There is a pool of potential residents of the community of size 1. These can be thought of as households who for exogenous reasons (employment opportunities, family ties, etc) need to live in the area in which the community is located and are potentially open to living in the community. Potential residents are characterized by their desire to live in the community (as opposed to an alternative community in the area) which is measured by the preference parameter θ. This desire, for example, may be determined by a household's idiosyncratic reaction to the community's natural amenities. The preference parameter takes on values between 0 and $\overline{\theta}$, and the fraction of potential residents with preference below $\theta \in [0, \overline{\theta}]$ is $\theta/\overline{\theta}$. Reflecting the fact that households' circumstances change over time, in each period new households join the pool of potential residents and old ones leave. The probability that a household currently a potential resident will be one in the subsequent period is $\mu \in (0, 1)$. Thus, in each period, a fraction $1 - \mu$ of households leave the pool and are replaced by an equal number of new ones.

The only way to live in the community is to own a house. The community has sufficient land to accommodate housing for all the potential residents. Moreover, the only use for land is building houses.[8] Houses are infinitely durable and the cost of building a new one is C.[9] Housing is supplied by competitive developers. The stock of houses at the beginning of a period is denoted by H and the stock at the beginning of the next period by H'. New construction is therefore $H' - H$. A stock of housing H can accommodate a fraction H of the pool of potential residents. The initial housing stock is denoted H_0.[10]

[7] The underlying economic model is a single community version of that presented by Barseghyan and Coate (2016), amended to include infinitely durable housing, a congestible durable local public good, and debt. A similar model is employed by Coate and Ma (2017) in their critique of using housing price changes to evaluate public investment, although they assume a fixed housing stock and no debt.

[8] We could alternatively assume that land not used for housing has a constant productivity in agricultural use.

[9] The assumption of infinitely durable housing is common in the urban economics literature and is justified by the fact that buildings in developed countries display considerable longevity. See Brueckner (2000) for more discussion of the different modelling assumptions used in the literature.

[10] It is important that H_0 be positive, so the community has residents. If this were not the case, there would be nobody to choose the period 0 policies. This assumption contrasts with Henderson (1980) who considers the development of an empty community by a developer. In future work, it would be interesting to add an additional initial development stage managed by developers to the model of this paper. One could then try to model the transition of political control from developers to residents. See Chapter 7 of Fischel (2015) for discussion of this process and Knapp (1991) for a model focusing on the timing of the transfer of control of the maintenance of a durable public good from a developer to residents in a condominium setting.

The community provides a durable local public good which depreciates at rate $\delta \in (0,1)$. The good costs c per unit. The stock at the beginning of a period is denoted by g and the stock at the beginning of the next period by g'. Given depreciation, the public good level available during the period is $g'/(1-\delta)$ and investment is $g'/(1-\delta)-g$.[11] The initial public good level is denoted g_0.

When living in the community, households have preferences defined over the public good and consumption. A household with preference parameter θ and consumption x obtains a period payoff of $\theta + x + B([g'/(1-\delta)]\,/\,(H')^{\alpha})$ from living in the community if the public good level is $g'/(1-\delta)$ and the number of households is H'.[12] The public good benefit function B is increasing, smooth, strictly concave, and satisfies the limit conditions that $lim_{z\searrow 0}B'(z)=\infty$ and $lim_{z\nearrow\infty}B'(z)=0$. The parameter α measures the congestibility of the public good and belongs to the interval $[0,1]$. The smaller is α, the closer the good is to a pure public good. The higher is α, the closer the good is to a publicly provided private good. When not living in the community, a household's per period payoff is $\underline{u}$.[13] Households discount future payoffs at rate β and can borrow and save at rate $\rho = 1/\beta - 1$. This assumption means that households are indifferent to the intertemporal allocation of their consumption. Each household in the pool receives an exogenous income stream, the present value of which is sufficient to pay taxes and purchase housing in the community.[14]

A competitive housing market operates in each period. Demand comes from new households moving into the community, while supply comes from owners leaving the community and new construction. The price of houses is denoted P. This price can fall below the construction cost C when demand at C is less than the stock H.

The community can also borrow and save at rate ρ. The community's debt level at the beginning of a period is denoted by b and the level at the beginning of the next period by b'. The community levies a tax T which is paid by all households who reside in the community at the end of the period. The community's budget constraint is therefore

$$(1+\rho)b + c\left(\frac{g'}{1-\delta} - g\right) = b' + H'T. \tag{1}$$

[11] Disinvestment is possible, so that $g'/(1-\delta)$ can be smaller than g. This assumption is made for the purposes of tractability. While unrealistic for durable public goods like roads, the reader can be assured that disinvestment does not occur in the equilibrium we study.

[12] We do not distinguish between the size of the housing stock and the number of residents since these will be the same in equilibrium.

[13] Note that $\underline{u}$ is both the per period payoff of living in one of the other communities in the geographic area if a household is in the pool and the payoff from living outside the area when a household leaves the pool.

[14] The assumption that utility is linear in consumption means that there are no income effects, so it is not necessary to be specific about the income distribution.

The left hand side is government spending and consists of debt repayment and investment. The right hand side is government revenues and consists of new borrowing and tax revenues. The community's initial debt level is denoted b_0.

The timing of the model is as follows. Each period, the community starts with a public good level g, a debt level b, and a stock of houses H. At the beginning of the period, the existing residents choose the level of investment in the public good, how to adjust the community's debt position, and how much tax to levy. This determines g', b', and T. Then, households who were in the pool of potential residents in the previous period learn whether they will be remaining and new households join. Those in the pool decide whether to live in the community and existing residents no longer in the pool prepare to leave it. The housing market opens and the equilibrium price P is determined along with new construction or, equivalently, next period's housing stock H'. New residents buy houses and move into the community and old ones sell up and leave. Residents enjoy public good benefits $B([g'/(1-\delta)]\,/\,(H')^{\alpha})$ and pay taxes T. The policies must satisfy the community's budget constraint. Next period begins with the state (g', b', H').

3.1 Housing market

We now explain in more detail how the housing market determines price and new construction. Note first that, at the beginning of any period, households fall into two groups: those who resided in the community in the previous period and those who did not, but could in the current period. Households in the first group own homes, while the second group do not. Households in the first group who leave the pool sell their houses and obtain a continuation payoff of

$$P + \frac{u}{1-\beta}. \tag{2}$$

The remaining households in the first group and all those in the second must decide whether to live in the community. This decision will depend on their preference parameter θ, current and future housing prices, and expected public good provision and taxes. Since selling a house and moving is costless, there is no loss of generality in assuming that all households sell their property at the beginning of any period.[15] This makes each household's location decision independent of its property ownership state. It also means that the only future consequences of the current location choice is through the price of housing in the next period.

[15] It should be stressed that this is just a convenient way of understanding the household decision problem. The equilibrium we study is perfectly consistent with the assumption that the only households selling their homes are those who plan to leave the community.

Let $P(g', b', H')$ denote the anticipated equilibrium price of housing when the state is (g', b', H'). Then, in a period in which the initial state is (g, b, H), the price of housing is P, and households anticipate $g'/(1-\delta)$ units of the public good to be provided, H' households to live in the community, and a tax of T, a household of type θ will choose to reside in the community if and only if

$$\theta + B\left(\frac{g'/(1-\delta)}{(H')^{\alpha}}\right) - T - P + \beta P(g', b', H') \geq \underline{u}. \tag{3}$$

The left hand side of this inequality represents the per-period payoff from locating in the community, assuming that the household buys a house at the beginning of the period and sells it the next. This payoff depends on the preference parameter θ, public good surplus, and current and future housing prices. The right hand side represents the per-period payoff from living elsewhere.

Given (3) and the fact that household preferences are uniformly distributed over $[0, \overline{\theta}]$, the equilibrium price of housing P in the current period must satisfy the market clearing condition

$$H' = 1 - \frac{\underline{u} - \left(B\left(\frac{g'/(1-\delta)}{(H')^{\alpha}}\right) - T - P + \beta P(g', b', H')\right)}{\overline{\theta}}. \tag{4}$$

This implies that the equilibrium price is

$$P = (1 - H')\overline{\theta} + B\left(\frac{g'/(1-\delta)}{(H')^{\alpha}}\right) - T + \beta P(g', b', H') - \underline{u}. \tag{5}$$

On the supply side, the assumption that houses are infinitely durable implies that

$$H' \geq H. \tag{6}$$

Moreover, because the supply of new construction is perfectly elastic at a price equal to the construction cost C, it must also be the case that

$$P \leq C \ (= \text{if } H' > H). \tag{7}$$

Given that next period's housing price is described by the function $P(g', b', H')$, any policy (g', b', T, H', P) which satisfies the budget constraint (1), is consistent with housing market equilibrium if and only if (5), (6), and (7) are satisfied.[16]

[16] This formulation implicitly assumes that P can, in principle, be negative. This is unrealistic because, in reality, residents can simply abandon their houses and leave the community if they so choose. However, imposing the constraint that the equilibrium housing price must be non-negative creates some additional complications that are inessential. In particular, it requires that we introduce a community debt limit. Such a limit is needed to prevent current residents from borrowing a large amount, using it to finance transfers to residents, and then abandoning the community and its debt the next period.

3.2 Policy choice

Next we turn to residents' choice of policies. As explained above, the timing of the model is first that the existing residents choose fiscal policies, and then the housing market determines new construction and the price of housing. Obviously, when residents choose policies they will anticipate how they impact the housing market. Rather than deriving the relationship between the housing market equilibrium and the policies, and then analyzing the optimal policies, it is easier to think of residents as directly choosing the housing price and new construction along with the policies, but subject to the constraint that their choice be consistent with housing market equilibrium. Thus, given any initial state (g, b, H), we assume that residents choose (g', b', T, H', P) but subject to the market equilibrium constraints (5), (6), and (7) along with the community budget constraint (1).

While residents differ in their desires to live in the community θ, they will have identical preferences over policies and hence there is no collective choice problem to resolve. To understand this, note that given the initial housing stock H, existing residents will have preferences in the interval $[(1-H)\overline{\theta}, \overline{\theta}]$.[17] We know that the market will allocate housing to those in the pool of potential residents with the highest θ and that the supply of housing can only expand. It follows that residents will all anticipate living in the community as long as they stay in the pool. In light of this, the residents' policy problem can be written as

$$\max_{(g',b',T,H',P)} \left\{ \begin{array}{c} (1-\mu)\left[P + \frac{u}{1-\beta}\right] + \mu\left[B\left(\frac{g'/(1-\delta)}{(H')^{\alpha}}\right) - T + \beta V(g', b', H')\right] \\ s.t.\ (1),\ (5),\ (6),\ \&\ (7) \end{array} \right\}. \tag{8}$$

This reflects the fact that, with probability $1-\mu$, a resident leaves the pool and sells its house, and, with probability μ, it remains in the pool and continues to live in the community. Note that $V(g', b', H')$ can be interpreted as the continuation payoff of a household with preference parameter $\theta = 0$ at the beginning of a period in which: the initial state is (g', b', H'); the household owns a house in the community but does not know whether it will remain in the pool; and the household is constrained to live in the community as long as it remains in the pool.

3.3 Equilibrium defined

We can now formally define an equilibrium of the model. An *equilibrium* consists of an investment rule $g'(g, b, H)$, a debt rule $b'(g, b, H)$, a tax rule $T(g, b, H)$, a housing rule $H'(g, b, H)$, a price rule

[17] In periods $t = 1, ..., \infty$ this follows from the fact that, in equilibrium, the households with the highest preference for living in the community purchase houses in the community in the previous period. We assume that this condition also characterizes the initial distribution of residents in period 0.

$P(g, b, H)$, and a value function $V(g, b, H)$ satisfying two conditions. First, for all states (g, b, H), the policies solve problem (8) given that the continuation value is described by $V(g', b', H')$ and the future housing price by $P(g', b', H')$. Second, for all (g, b, H), the value function satisfies the equality

$$V(g, b, H) = (1 - \mu)\left[P(\cdot) + \frac{\underline{u}}{1-\beta}\right] + \mu\left[B\left(\frac{g'(\cdot)/(1-\delta)}{H'(\cdot)^{\alpha}}\right) - T(\cdot) + \beta V(g'(\cdot), b'(\cdot), H'(\cdot))\right], \tag{9}$$

where $g'(\cdot)$ denotes the policy $g'(g, b, H)$, etc.

4 Optimal community development

To evaluate the normative performance of community development by public wealth accumulation, we need a benchmark for comparison. This section characterizes the development plan that would be optimal for a Utilitarian planner. Such a planner maximizes the discounted sum of the aggregate payoffs of the different pools of potential residents. The assumption that utility is linear in consumption, implies that the planner is indifferent between transfers of consumption both between households in the same pool and across different pools. Accordingly, there is no loss of generality in simply assuming that, in any period, the cost of new construction and investment is financed by lump-sum taxation of the pool of potential residents.

The planner's problem can be posed recursively. Given initial stocks of public good and housing (g, H), the planner chooses investment and new construction or, equivalently, next period's public good and housing stock (g', H'). The planner will allocate the households in the pool with the highest θ to the H' houses. Given that θ is uniformly distributed on $[0, \overline{\theta}]$, this implies that households in the interval $[(1 - H')\,\overline{\theta}, \overline{\theta}]$ will be assigned to live in the community. Accordingly, the planner's problem is

$$U(g, H) = \max_{(g', H')}\left\{\begin{array}{c} \int_{(1-H')\overline{\theta}}^{\overline{\theta}} \theta \frac{d\theta}{\overline{\theta}} + H'B\left(\frac{g'/(1-\delta)}{(H')^{\alpha}}\right) + \underline{u}\,(1 - H') - C(H' - H) \\ -c(g'/(1-\delta) - g) + \beta U(g', H') \\ s.t.\ (6) \end{array}\right\}. \tag{10}$$

The first two terms in the objective function represent the benefits received by the households assigned to the community, while the third term represents the benefits to those not so assigned. The fourth and fifth terms represent, respectively, the costs of new construction and investment. The final term is the continuation value.

Observe that g enters the value function linearly, so that $\partial U(g,H)/\partial g$ is just equal to c. Given this, it is straightforward to verify that g' must equal $(1-\delta)g^o(H')$ where $g^o(H)$ satisfies the dynamic Samuelson rule

$$H^{1-\alpha}B'\left(\frac{g^o}{H^\alpha}\right) = c(1-\beta(1-\delta)). \tag{11}$$

The left hand side measures the per-period social benefit of an additional unit of public good and the right hand side the per-period cost. The latter reflects the fact that a fraction $1-\delta$ of a unit purchased this period will be available for use next period.

To characterize the optimal level of housing in a way that makes it comparable with what happens in equilibrium, it is convenient to introduce the function $S(H)$ which represents per-resident *optimized public good surplus*, defined as

$$S(H) \equiv B\left(\frac{g^o(H)}{H^\alpha}\right) - \frac{c(1-\beta(1-\delta))g^o(H)}{H}. \tag{12}$$

This surplus is the difference between the public good benefits enjoyed by each resident at the optimal level and the per-resident cost of this level computed using the per-period marginal cost from (11). Using this function, we can substitute out the public good and rewrite the planner's problem as follows:

$$U(H) = \max_{H'} \left\{ \begin{array}{c} \int_{(1-H')\overline{\theta}}^{\overline{\theta}} \theta \frac{d\theta}{\overline{\theta}} + H'S(H') + \underline{u}\,(1-H') - C(H'-H) \\ +c(1-\delta)(g^o(H) - \beta g^o(H')) + \beta U(H') \\ s.t.\ H' \geq H \end{array} \right\}. \tag{13}$$

Maximizing with respect to H' and using the envelope condition that $U'(H')$ is equal to $C + c(1-\delta)dg^o(H')/dH$ reveals that the optimal level of housing H^o satisfies the first order condition[18]

$$(1-H^o)\,\overline{\theta} + S(H^o) + H^o S'(H^o) - C(1-\beta) = \underline{u}. \tag{14}$$

The left hand side represents the net social benefit from assigning an additional household to the community. The first term is the preference of the marginal household for living in the community; the second, the optimized public good surplus accruing to the marginal household; the third, the impact of adding the household on the public good surpluses of the other residents; and the fourth, the per-period cost of an additional house. The optimal housing level is such that this net social

[18] This assumes that the constraint $H' \geq H$ is not binding.

benefit is just equal to the benefit the household receives when not residing in the community, $\underline{u}$. Note that

$$HS'(H) = \frac{(1-\alpha)c(1-\beta(1-\delta))g^{o}(H)}{H}, \tag{15}$$

so that the impact on other residents' public good surpluses of adding a household is always positive provided that α is less than 1. This reflects the benefits of sharing the costs of the public good.

We impose the following assumption to make sure that the planner's problem is well-behaved.

Assumption 1 (i) *For all $H \in [H_0, 1]$*

$$-\overline{\theta} + 2S'(H) + HS''(H) < 0.$$

(ii)

$$(1-H_0)\,\overline{\theta} + S(H_0) + H_0S'(H_0) - C(1-\beta) > \underline{u} > S(1) + S'(1) - C(1-\beta).$$

Part (i) of the assumption implies that net social benefit from assigning an additional household to the community is decreasing in the number of households. Part (ii) implies that this net social benefit exceeds $\underline{u}$ at the initial population H_0 but falls below it when the entire pool lives in the community. Together, the two parts imply that there exists a unique solution to the first order condition (14) that lies between H_0 and 1. This solution unambiguously defines the optimal housing level. We may therefore conclude:

Proposition 1 *Under Assumption 1, the optimal community development plan is to construct $H^{o} - H_0$ new houses in period 0 and invest in $g^{o}(H^{o}) - g_0$ units of the public good. Thereafter, no more housing should be constructed and the public good level should be maintained at $g^{o}(H^{o})$.*

There are three main points to take away about the optimal plan. First, development occurs immediately. Second, the public good satisfies the dynamic Samuelson rule. Third, the size of the community balances the net social benefit of an additional household to the payoff households get from living elsewhere.

5 Finding equilibrium

This section is devoted to finding an equilibrium of the model. Readers just interested in seeing what happens in this equilibrium can skip ahead to Section 6. As discussed in the introduction, the strategy for finding equilibrium in this type of model is "guess and verify". To develop intuition for what to guess, we start by characterizing the development plan that would be optimal for the initial residents of the community (i.e., those owning houses at the beginning of period 0).

5.1 The initial residents' optimal plan

Suppose that the initial residents could commit the community to following a complete development plan $\{g_{t+1}, b_{t+1}, T_t, H_{t+1}, P_t\}_{t=0}^{\infty}$. Here g_{t+1} denotes the level of the public good at the beginning of period $t+1$, b_{t+1} the level of debt at the beginning of period $t+1$, etc. Their optimal plan would maximize the objective function

$$\sum_{t=0}^{\infty} (\mu\beta)^t \left\{ (1-\mu)\left[P_t + \frac{\underline{u}}{1-\beta}\right] + \mu\left[B\left(\frac{g_{t+1}/(1-\delta)}{(H_{t+1})^{\alpha}}\right) - T_t\right]\right\}, \tag{16}$$

subject to in each period t satisfying the budget constraint (1) and the constraints of market equilibrium (5), (6), and (7).[19]

Before we can describe the solution, we need a little more notation and two additional assumptions. First, let W denote the community's wealth (i.e., $W = cg - (1+\rho)b$). Wealth at the beginning of period t is denoted W_t. Next, for all wealth levels for which it is well defined, let $\mathcal{H}(W)$ be the largest housing level in the interval $[0, 1]$ that satisfies the equality

$$(1-H)\,\overline{\theta} + S(H) + \frac{(1-\beta)W}{H} - C(1-\beta) = \underline{u}. \tag{17}$$

To interpret this housing level, note that $S(H)+(1-\beta)W/H$ is the public good surplus a household would enjoy if the community had H residents, a wealth level W, and provides the efficient public good level, financing provision so as to keep wealth constant.[20] Accordingly, the expression on the left hand side of the equality, represents the per-period benefit that the marginal home buyer would obtain from living in the community under these conditions, assuming the price of housing is constant at C. It follows that $\mathcal{H}(W)$ is the largest population the community can attract when it has wealth W, assuming it provides the efficient public good level and finances provision so as to keep wealth constant.

Note that $\mathcal{H}(W)$ will not be well-defined if W is so large that all types of households would get a positive benefit from living in the community or if W is so small (i.e., sufficiently negative) that there is no population size at which residents would be willing to pay a price of housing C to live in the community. On the interval of wealth levels on which it is well-defined, $\mathcal{H}(W)$ will

[19] We also need to add the standard transversality condition that $\lim_{t\to\infty} \beta^t b_t = 0$ to rule out the initial residents operating a Ponzi scheme.

[20] Public good surplus if the community has H residents and provides the efficient level of the public good is $B\left(g^o(H)/H^{\alpha}\right) - T$. Using (1) and the fact that $cg - (1+\rho)b$ is equal to W, the tax will equal $\left[cg^o(H) - W - b'\right]/H$. If the community's wealth is constant at W, this means that $c(1-\delta)g^o(H) + (1+\rho)b'$ is equal to W. Solving this equation for b' and using the fact that $1+\rho$ is equal to $1/\beta$, we can write the tax as $\left[cg^o(H)(1-\beta(1-\delta)) - W(1-\beta)\right]/H$. The claim now follows from the definition of $S(H)$ in (12).

be increasing under Assumption 1. Intuitively, the community will be more attractive to potential residents with higher wealth because it offers more public good surplus. Indeed, this is the essence of the fiscal externality created by the collective ownership of the community's assets. It is also the case that $\mathcal{H}(W)$ will be concave under Assumption 1.[21] Finally, note that because $\mathcal{H}(W)$ is increasing on the interval of wealth levels on which it is well-defined, it has an inverse. This function, which we denote $\mathcal{W}(H)$, tells us the wealth the community needs to attract a population of size H when it provides the efficient public good level, financing provision so as to keep wealth constant, and the price of housing is constant at C. In light of the properties of $\mathcal{H}(W)$, $\mathcal{W}(H)$ will be increasing and convex.

Our assumptions concern the community's initial conditions. The first is that the community's initial housing stock H_0 satisfies:

Assumption 2

$$\overline{u} + H_0\overline{\theta} > (1 - H_0)\,\overline{\theta} + S(H_0) + H_0 S'(H_0) - C(1-\beta) > \underline{u} + H_0\overline{\theta}\left(1 - \frac{\mu^2(1-\beta)}{1-\mu\beta}\right).$$

The first inequality in this Assumption guarantees that the interval on which the function $\mathcal{W}(H)$ is well-defined includes all the housing levels that could in principle arise in equilibrium (i.e., those in the interval $[H_0, 1]$). The role of the second inequality will be explained in the next sub-section. The second assumption concerns the community's initial wealth W_0:

Assumption 3 *The community's initial public good level g_0 and debt level b_0 are such that*

$$W_0 < \mathcal{W}(1).$$

This assumption ensures that the community does not start out with so much wealth that all types of households would want to live in it, if the efficient public good level were provided, provision financed so as to keep wealth constant, and the price of housing were constant at C.

We can now describe the initial residents' optimal plan.

Proposition 2 *If Assumptions 1-3 are satisfied, there exist wealth levels $W^*(H_0)$ and $W_n(H_0)$, satisfying $\mathcal{W}(H_0) < W^*(H_0) < W_n(H_0)$, such that under the initial residents' optimal plan:*
(i) If $W_0 \geq W^*(H_0)$, the community invests in $g^o(\mathcal{H}(W_0)) - g_0$ units of the public good in period 0 *and the market provides $\mathcal{H}(W_0) - H_0$ new houses. The community finances investment so as to keep its wealth constant, meaning that all but $c\delta g^o(\mathcal{H}(W_0))$ is financed with debt. Thereafter, the*

[21] The properties of $\mathcal{H}(W)$ are established in Appendix 0.

public good is maintained at $g^o(\mathcal{H}(W_0))$ and no more housing is provided. Taxes are set so that wealth remains at W_0. Throughout, the price of housing is C.

(ii) If $W_0 < W^*(H_0)$, the community invests in $g^o(H_0) - g_0$ units of the public good in period 0 and the market provides no new houses. The community chooses debt and taxes so that its wealth increases to $W_n(H_0)$. In period 1, the community invests in $g^o(\mathcal{H}(W_n(H_0))) - (1-\delta)g^o(H_0)$ units of the public good and the market provides $\mathcal{H}(W_n(H_0)) - H_0$ new houses. Investment is financed so as to keep wealth constant, implying that all but $c\delta g^o(\mathcal{H}(W_n(H_0)))$ is financed with debt. Thereafter, the public good is maintained at $g^o(\mathcal{H}(W_n(H_0)))$ and no more housing is provided. Taxes are set so that wealth remains at $W_n(H_0)$. The price of housing is less than C in period 0 and C thereafter.

Thus, the initial residents' optimal plan takes one of two forms. Under the first, anticipating an increase in population, residents invest in the public good in the initial period, and the market provides new construction. The residents finance the increase in the stock of the public good entirely with debt, keeping the community's wealth constant. Thereafter, there is no further development. This means that the size of the community grows to $\mathcal{H}(W_0)$ which is the largest population that it can attract with its initial wealth if it provides the public good efficiently. Under the second form, the community accumulates wealth in the initial period and no development takes place. The motivation for accumulating wealth is to spur development in the next period. Thereafter, things follow the pattern of the first form. However, the size of the community grows to $\mathcal{H}(W_n(H_0))$ which is the largest population it can attract with its new higher wealth level.

The optimal plan involves wealth accumulation when the community has low initial wealth. Here, low is defined relative to the endogenous threshold wealth level $W^*(H_0)$. Notice that the proposition tells us that this threshold wealth level $W^*(H_0)$ exceeds $\mathcal{W}(H_0)$. This means that when the community's initial wealth W_0 lies between $\mathcal{W}(H_0)$ and $W^*(H_0)$, the initial residents choose to accumulate wealth even though some development is possible without accumulation (since W_0 being larger than $\mathcal{W}(H_0)$ implies that $\mathcal{H}(W_0)$ exceeds H_0). They do this, because the sacrifice in the initial period in terms of higher taxes and a smaller population, is compensated by the benefits of a larger population in the future. Effectively, the initial residents find it profitable to leverage the fiscal externality created by the collective ownership of the community's assets to increase the extent of overall development.

The proof of this proposition, along with those of Propositions 3 and 4, is in Appendix 1. The initial residents' problem involves a lot of choice variables and many constraints which makes deriving the solution challenging. Despite this, the solution is simple and intuitive. The only possibly counter-intuitive feature is that, when the community accumulates, all the accumulating is done in the initial period rather than being spread over time. One might have guessed that the initial residents would delay some of the accumulation until they had attracted more residents, thereby spreading the burden of accumulation over a larger population. However this turns out not to be desirable because such a strategy would delay entry into the community. Any accumulation to which potential residents have to contribute is anticipated and reduces the set of potential residents willing to join the community.[22]

5.2 Time consistency

We now look at whether future residents will wish to follow the initial residents' optimal plan. This will provide insight into how the initial residents will need to modify their initial period policy choices when decision-making is sequential.

Using standard terminology, the initial residents' optimal plan $\{g_{t+1}, b_{t+1}, T_t, H_{t+1}, P_t\}_{t=0}^{\infty}$ is *time consistent* if, for all $t \geq 1$, $\{g_{\tau+1}, b_{\tau+1}, T_\tau, H_{\tau+1}, P_\tau\}_{\tau=t}^{\infty}$ is an optimal plan for those residents in the community at the beginning of period t, given the initial condition (g_t, b_t, H_t). To assess time consistency, we need to understand what optimal plans for future residents look like. The optimal plan for the period t residents will solve the same problem as for the period 0 residents, except that the community's wealth and housing stock will be (W_t, H_t) rather than (W_0, H_0). The housing level H_t could be any level in the interval $[H_0, 1]$. We assume that W_t is less than $\mathcal{W}(1)$, since given Assumption 3 and Proposition 2 there is no need to consider higher levels.

To describe the period t residents' optimal plan, we need to introduce the housing level H^s which is implicitly defined by the equation

$$(1 - H^s)\,\overline{\theta} + S(H^s) + H^s S'(H^s) - C(1-\beta) = \underline{u} + H^s\overline{\theta}\left(1 - \frac{\mu^2(1-\beta)}{1-\mu\beta}\right). \tag{18}$$

This equation is quite similar to that which defines the optimal housing level H^o in (14) but contains an additional term on the right hand side. Assumption 1 and the second inequality in Assumption 2 imply that H^s is well-defined and lies between H_0 and H^o.[23] The nature of the

[22] To be concrete, imagine that the initial residents were to reduce wealth accumulation from $W_n(H_0)$ to $W_n(H_0) - \epsilon$ in the initial period, where ϵ is small and positive, and make up the difference in period 1. This would decrease the population from $\mathcal{H}(W_n(H_0))$ to $\mathcal{H}(W_n(H_0) - \epsilon/(1-\beta))$ in period 1 which would translate into a lower level of public good surplus in period 1 for initial residents. The benefit would be to reduce taxes by ϵ/H_0 in the initial period. It can be shown that if the period 1 loss of public good surplus were less than the initial period benefit from lower taxes, then it would be even better to not make up the difference in period 1 and simply reduce the total amount of accumulation to $W_n(H_0) - \epsilon$. This contradicts the fact that $W_n(H_0)$ is the optimal amount of wealth to accumulate.

[23] The second inequality in Assumption 2 implies that the left hand side of equation (18) exceeds the right hand

optimal plan depends on whether H_t is smaller or larger than H^s. In the former case, the optimal plan is similar to that of the initial residents.

Proposition 3 *If $H_t \in [H_0, H^s)$, Assumptions 1 and 2 are satisfied, and $W_t < \mathcal{W}(1)$, there exist wealth levels $W^*(H_t)$ and $W_n(H_t)$, satisfying $\mathcal{W}(H_t) < W^*(H_t) < W_n(H_t)$, such that the period t residents' optimal plan has the form described in Proposition 2. Moreover, the functions $W^*(H_t)$ and $W_n(H_t)$ are differentiable and increasing on $[H_0, H^s)$, and satisfy the limit condition that $\lim_{H_t \nearrow H^s} W^*(H_t) = \lim_{H_t \nearrow H^s} W_n(H_t) = \mathcal{W}(H^s)$.*

The only noteworthy point is that both $W^*(H_t)$ and $W_n(H_t)$ are increasing in H_t. This implies that, for housing levels in the interval $[H_0, H^s)$, the residents of a larger community are more willing to choose public wealth accumulation and do more accumulation (in an absolute sense) when they do so. This reflects the fact that the costs of such accumulation are spread over a larger population.

The case in which H_t is larger than H^s is more interesting.

Proposition 4 *If $H_t \in [H^s, 1]$, Assumptions 1 and 2 are satisfied, and $W_t < \mathcal{W}(1)$, then under the period t residents' optimal plan:*

(i) If $W_t > \mathcal{W}(H_t)$, the community invests in $g^o(\mathcal{H}(W_t)) - g_t$ units of the public good in period t and the market provides $\mathcal{H}(W_t) - H_t$ new houses. The community finances investment so as to keep its wealth constant, meaning that all but $c\delta g^o(\mathcal{H}(W_t))$ is financed with debt. Thereafter, the public good is maintained at $g^o(\mathcal{H}(W_t))$ and no more housing is provided. Taxes are set so that wealth remains at W_t. Throughout, the price of housing is C.

(ii) If $W_t \leq \mathcal{W}(H_t)$, the market provides no new houses and the size of the community remains at H_t. The community invests in $g^o(H_t) - g_t$ units of the public good in period t and, thereafter, the public good is maintained at $g^o(H_t)$. The way in which the community finances investment is not tied down, but wealth remains no greater than $\mathcal{W}(H_t)$. One possibility is that wealth increases to $\mathcal{W}(H_t)$ and then remains there. In this case, the price of housing is less than C in the initial period and equal to C thereafter.

The key point to note is that when H_t is larger than H^s, the period t residents do not engage in public wealth accumulation to increase future development. Development will occur if the community's initial wealth is sufficient to attract in more residents but not otherwise. This reflects the concavity of the function $\mathcal{H}(W)$. To see this, suppose that W_t is less than $\mathcal{W}(H_t)$. There is no cost to the period t residents of increasing wealth to $\mathcal{W}(H_t)$, since this will not change the

side at housing level H_0. The right hand side is increasing in H and Assumption 1(i) implies that the left hand side is decreasing in H. Moreover, the left hand side is less than the right hand side at housing level H^o.

future size of the community. If the residents remain in the community they will benefit from the lower taxes its higher wealth allows and if they leave, the higher wealth will be capitalized into the price of housing. This offsets the tax cost of accumulation. Increasing wealth above $\mathcal{W}(H_t)$ will impact period t residents' payoffs. If the residents remain in the community they will share the benefit from its higher wealth with a larger population and if they leave, the higher wealth will not be capitalized into the price of housing which just equals the supply price C. The benefit of increasing the future size of the community via the fiscal externality must be weighed against the tax cost. The extent of the population increase stemming from marginally increasing wealth above $\mathcal{W}(H_t)$ will be governed by the derivative $\mathcal{H}'(\mathcal{W}(H_t))$. Given the concavity of $\mathcal{H}(W)$, this marginal benefit will be decreasing in $\mathcal{W}(H_t)$ and hence H_t. On the other hand, the marginal cost of accumulation is decreasing in H_t. The marginal benefit is falling faster than the marginal cost and the housing level H^s is where the marginal benefit equals the marginal cost. Beyond the housing level H^s, therefore, the marginal benefit is smaller than the marginal cost and increasing wealth beyond $\mathcal{W}(H_t)$ is not attractive.

Combining Propositions 2, 3, and 4, we can now establish:

Proposition 5 *If Assumptions 1-3 are satisfied, the initial residents' optimal plan is time consistent if and only if* $W_0 \geq \mathcal{W}(H^s)$.

To see why the "if" part of the Proposition is true, note that if W_0 exceeds $\mathcal{W}(H^s)$, then the initial residents' optimal plan implies that H_1 exceeds H^s and that W_1 equals $\mathcal{W}(H_1)$. Thereafter, housing and wealth are supposed to remain constant. This is time consistent because, by part (ii) of Proposition 4, if the period t residents have a housing stock H_t exceeding H^s and a wealth level $\mathcal{W}(H_t)$, they will indeed be happy to keep wealth and the future housing stock constant. For the "only if" part, if W_0 is less than $\mathcal{W}(H^s)$, then W_0 could either exceed or be smaller than the threshold $W^*(H_0)$. In the former case, new construction occurs in period 0 under the initial residents' optimal plan and (W_1, H_1) will equal $(W_0, \mathcal{H}(W_0))$. Thereafter housing and wealth remain constant. However, H_1 will be less than H^s and W_1 will equal $\mathcal{W}(H_1)$ and hence be strictly less than $W^*(H_1)$. By Proposition 3, the period 1 residents will want to increase taxes and accumulate wealth to attract new residents. In the latter case, new construction does not occur until period 1 and it is the period 2 residents who want to deviate. Under the initial residents' optimal plan, at the beginning of period 2, (W_2, H_2) will equal $(W_n(H_0), \mathcal{H}(W_n(H_0)))$ and thereafter housing and wealth are supposed to remain constant. However, H_2 will be less than H^s and W_2 will equal $\mathcal{W}(H_2)$, so the period 2 residents will want to increase taxes and accumulate wealth.

Proposition 5 suggests that when decision-making is sequential, the initial residents will need to adjust their period 0 policy choices unless initial wealth exceeds $\mathcal{W}(H^s)$. In particular, in the case in which initial wealth W_0 lies between the threshold $W^*(H_0)$ and $\mathcal{W}(H^s)$, they will need to recognize that simply investing in $g^o(\mathcal{H}(W_0)) - g_0$ units of the public good in period 0 and financing investment so as to keep wealth constant, will not result in the community increasing in size to $\mathcal{H}(W_0)$. This is because with initial wealth W_0 and housing stock $\mathcal{H}(W_0)$, the period 1 residents will wish to accumulate more public assets. This will imply a reduction in the price of housing in period 1, the anticipation of which will drive the equilibrium price in period 0 below the supply price C. To prevent this from happening, the initial residents will need to tax-finance some investment so as to carry forward more wealth. However, this will necessitate a reduction in the amount of development as potential residents are deterred by the higher taxes.

When guessing an equilibrium, we need to specify the policy choices residents will make for any given initial conditions - not just those facing the initial residents. In particular, we need to worry about what happens for housing levels higher than H_0. For this purposes, it is useful to note the following generalization of Proposition 5:

Proposition 6 *If Assumptions 1-3 are satisfied, the period t residents' optimal plan is time consistent if and only if either $W_t \geq \mathcal{W}(H^s)$ or $H_t \geq H^s$.*

This result follows immediately from Proposition 4 since the optimal plan for a community with a housing level H_t larger than H^s and wealth smaller than $\mathcal{W}(H^s)$ leaves next period's residents with a housing level H_t and wealth no greater than $\mathcal{W}(H_t)$. By Proposition 4, this is a situation they have no incentive to change. Proposition 6 suggests that when decision-making is sequential and the community has a housing stock at least as big as H^s, residents will make the same choices as they do in the initial period of the commitment solution.

5.3 The conjectured equilibrium

On the basis of all this, we can now make a guess for what equilibrium might look like. Our guess has four components. The first is that investment in the public good will always be efficient for the size of the community. This is convenient because it basically reduces the state variables from three to two - community wealth W and housing H. Given any target for next period's wealth, debt adjusts to meet it, under the assumption that the public good is set it to its efficient level.

The second component is that when H is greater than equal to H^s, residents will choose the policies from the initial period of their optimal plan. This is because Proposition 6 tells us that their optimal plan is time consistent. Specifically, what this means is that if W is larger than $\mathcal{W}(H)$, then the population increases to $\mathcal{H}(W)$ and the residents finance the investment necessary to service the new population to keep wealth constant. If W is less than or equal to $\mathcal{W}(H)$, then the population remains the same. From Proposition 4, there is not a uniquely optimal choice of wealth in the optimal plan. For the sake of concreteness, we assume that the residents increase wealth to $\mathcal{W}(H)$.[24] This choice makes future housing prices equal to C.

The third component of the guess is that when H is less than H^s and W is greater than or equal to $\mathcal{W}(H^s)$, residents will choose the policies from the initial period of their optimal plan. Again, this is because Propositions 5 and 6 tell us their optimal plan is time consistent. Thus, the population increases to $\mathcal{H}(W)$ and the residents finance the investment necessary to service the new population to keep wealth constant.

The fourth component concerns what happens when H is less than H^s and W is less than $\mathcal{W}(H^s)$. This is the most complicated case and our guess here has multiple sub-components. Based on Proposition 4, we guess there will exist a *threshold wealth function*, which we denote $W^*(H)$, such that new construction will occur if the community's wealth is greater than or equal to this threshold.[25] If the community's wealth is less than the threshold, there will be no new construction and the community will accumulate wealth. Based on Proposition 3, we conjecture this function will be increasing on $[H_0, H^s)$, will exceed $\mathcal{W}(H)$, and will satisfy the limit condition $\lim_{H \nearrow H^s} W^*(H) = \mathcal{W}(H^s)$.

When wealth is above the threshold and new construction occurs, we guess that the community's policy choices will not leave next period's residents with an incentive to accumulate wealth. Were they to do so, the fall in house prices caused by the higher taxes would be anticipated and the market deterred from providing new construction in the current period. Leaving next period's residents with no incentive to build wealth requires that the new state (W', H') is such that W' is at least as big as $W^*(H')$. Since there would seem to be no gain to current residents from leaving more wealth than necessary, we conjecture that W' will equal $W^*(H')$. Under this assumption,

[24] Note that this will not be categorized as development by public wealth accumulation, because there is no development.

[25] It should be clear that this threshold wealth function $W^*(H)$ will be related to, but not exactly the same as, the threshold wealth function $W^*(H_t)$ associated with Proposition 3. While a different notation could have been employed to more clearly distinguish the two functions, for the remainder of the paper $W^*(H)$ will refer to the threshold wealth function associated with the equilibrium, so no confusion should arise. Similar remarks apply to the function $W_n(H)$ discussed below.

equilibrium in the housing market will require that

$$(1-H')\overline{\theta}+S\left(H'\right)+\frac{W-\beta W^{*}(H')}{H'}-C(1-\beta)=\underline{u}. \tag{19}$$

This condition turns out to be sufficient to pin down what the new housing level H' must be.

When wealth is below the threshold, we guess there will exist some increasing function $W_n(H)$ describing how much the community will accumulate. This function will have the property that $W_n(H)$ exceeds $W^*(H)$, so that the wealth accumulation spurs development in the next period. The wealth level $W_n(H)$ will represent the optimal amount of accumulation for residents given the equilibrium play of future residents.

We have now described all four components of our guess. The key element is the threshold wealth function $W^*(H)$. Once we have this, the policy rules and value function follow from it. To see this, let Ψ denote the set of all real valued functions W^* defined on the interval $[H_0, H^s]$ with the properties that W^* is increasing, differentiable, and exceeds $\mathcal{W}$ on the interval $[H_0, H^s]$, and satisfies $W^*(H^s) = \mathcal{W}(H^s)$. For any $W^* \in \Psi$, we will show that we can define a corresponding candidate equilibrium, which we denote $\mathcal{E}(W^*)$.

To define this candidate equilibrium formally we just need to draw out the implications of the four components of our guess. We start with the public good. Given the first component of our guess, for all states (g, b, H), the public good rule is

$$g'(g,b,H)=(1-\delta)g^{o}(H'(g,b,H)). \tag{20}$$

We can also take care of the tax rule since this just follows from the budget constraint (1). Thus, for all states (g, b, H), the tax is

$$T(g,b,H)=\frac{(1+\rho)b+c\left(\frac{g'(g,b,H)}{1-\delta}-g\right)-b'(g,b,H)}{H'(g,b,H)}. \tag{21}$$

Next, we deal with states in which $H \in [H^s, 1]$. Given the second component of our guess, the housing rule is

$$H'(g,b,H)=\begin{cases} H & \text{if } W\leq\mathcal{W}(H) \\ \mathcal{H}(W) & \text{if } W\in(\mathcal{W}(H),\mathcal{W}(1)] \end{cases}, \tag{22}$$

and the debt rule is

$$b'(g,b,H)=\begin{cases} \frac{c(1-\delta)g^{o}(H)-\mathcal{W}(H)}{1+\rho} & \text{if } W\leq\mathcal{W}(H) \\ \frac{c(1-\delta)g^{o}(\mathcal{H}(W))-W}{1+\rho} & \text{if } W\in(\mathcal{W}(H),\mathcal{W}(1)] \end{cases}. \tag{23}$$

These rules imply that when W exceeds $\mathcal{W}(H)$ and new construction takes place, the community has wealth W at the beginning of the next period. When W is less than or equal to $\mathcal{W}(H)$, next period wealth is $\mathcal{W}(H)$. We also need to specify the price and value function for this part of the state space. In defining the price, we employ the notation

$$\mathcal{P}(H, W', W) = (1 - H)\overline{\theta} + S(H) + \frac{W - \beta W'}{H} + \beta C - \underline{u}, \tag{24}$$

to denote the price at which housing demand would equal H if wealth levels this and next period were W and W', the housing price next period were C, and the community provides the efficient level of the public good. Then we have

$$P(g, b, H) = \begin{cases} \mathcal{P}(H, \mathcal{W}(H), W) & \text{if } W \leq \mathcal{W}(H) \\ C & \text{if } W \in (\mathcal{W}(H), \mathcal{W}(1)] \end{cases}. \tag{25}$$

Note that, by definition, $\mathcal{P}(H, \mathcal{W}(H), \mathcal{W}(H))$ is equal to C, so the price will not exceed C. The value function in this part of the state space is

$$V(g, b, H) = \begin{cases} V^*(\mathcal{W}(H)) + \frac{W - \mathcal{W}(H)}{H} & \text{if } W \leq \mathcal{W}(H) \\ V^*(W) & \text{if } W \in (\mathcal{W}(H), \mathcal{W}(1)] \end{cases} \tag{26}$$

where on the interval $[\mathcal{W}(H^s), \mathcal{W}(1)]$ the function $V^*(W)$ is defined as

$$V^*(W) = C + \frac{\underline{u}}{1 - \beta} + \frac{\mu\overline{\theta}}{1 - \mu\beta}(\mathcal{H}(W) - 1). \tag{27}$$

It is worth noting that $V^*(W)$ is concave on the interval $[\mathcal{W}(H^s), \mathcal{W}(1)]$ given that $\mathcal{H}(W)$ is concave.

States in which $H \in [H_0, H^s)$ but the community's assets are such that $W \in [\mathcal{W}(H^s), \mathcal{W}(1)]$ are governed by the third component of our guess. Things are very simple for this case and we have that

$$(H'(g, b, H), b'(g, b, H), P(g, b, H)) = (\mathcal{H}(W), \frac{c(1 - \delta)g^o(\mathcal{H}(W)) - W}{1 + \rho}, C) \tag{28}$$

and that

$$V(g, b, H) = V^*(W), \tag{29}$$

where the function $V^*(W)$ is defined as in (27).

Finally, we deal with states in which $H \in [H_0, H^s)$ but the community's assets are such that W is less than $\mathcal{W}(H^s)$. This is the complicated case, covered by the fourth component of our guess.

It is here that the equilibrium policy rules depend on the particular threshold wealth function W^* we start with. The housing rule is

$$H'(g,b,H) = \begin{cases} H & \text{if } W < W^*(H) \\ H_c(W) & \text{if } W \in [W^*(H), \mathcal{W}(H^s)) \end{cases}, \tag{30}$$

where the function $H_c(W)$ is defined implicitly as the solution to the system:

$$(1-H_c)\overline{\theta} + S(H_c) + \frac{W - \beta W^*(H_c)}{H_c} - C(1-\beta) = \underline{u} \ \ \& \ \ W^*(H_c) \geq W. \tag{31}$$

It will be shown in the proof of the Theorem below that $H_c(W)$ is an increasing function on the interval $[W^*(H_0), \mathcal{W}(H^s))$ bounded above by H^s. The debt rule is

$$b'(g,b,H) = \begin{cases} \frac{c(1-\delta)g^o(H) - W_n(H)}{1+\rho} & \text{if } W < W^*(H) \\ \frac{c(1-\delta)g^o(H_c(W)) - W^*(H_c(W))}{1+\rho} & \text{if } W \in [W^*(H), \mathcal{W}(H^s)) \end{cases}, \tag{32}$$

where the function $W_n(H)$ will be defined formally below once the value function has been specified. These rules imply that when W exceeds $W^*(H)$ and new construction takes place, the community's wealth next period is the threshold level associated with its new housing stock $W^*(H_c(W))$. When W is smaller than $W^*(H)$, wealth next period is $W_n(H)$, so this represents the level the community will build up to when starting below the threshold. The price rule is

$$P(g,b,H) = \begin{cases} \mathcal{P}(H, W_n(H), W) & \text{if } W < W^*(H) \\ C & \text{if } W \in [W^*(H), \mathcal{W}(H^s)) \end{cases}. \tag{33}$$

The definition of $W_n(H)$ below will imply the price cannot exceed C. The value function is given by

$$V(g,b,H) = \begin{cases} V^*(W^*(H)) + \frac{W - W^*(H)}{H} & \text{if } W < W^*(H) \\ V^*(W) & \text{if } W \in [W^*(H), \mathcal{W}(H^s)) \end{cases}, \tag{34}$$

where on the interval $[W^*(H_0), \mathcal{W}(H^s))$ the function $V^*(W)$ is defined recursively as

$$V^*(W) = (1-\mu)\left[C + \frac{\underline{u}}{1-\beta}\right] + \mu\left[S(H_c(W)) + \frac{W - \beta W^*(H_c(W))}{H_c(W)} + \beta V^*(W^*(H_c(W)))\right]. \tag{35}$$

Finally, the function $W_n(H)$ is defined as

$$W_n(H) = \arg\max_{W'} \left\{ \begin{array}{c} (1-\mu)\left[\mathcal{P}(H, W', W^*(H)) + \frac{\underline{u}}{1-\beta}\right] + \mu\left[S(H) + \frac{W^*(H) - \beta W'}{H} + \beta V^*(W')\right] \\ s.t.\ \mathcal{P}(H, W', W^*(H)) \leq C \end{array} \right\}. \tag{36}$$

This maximization problem represents the problem faced by residents choosing next period's wealth under the assumption that there is no new construction this period. Notice that the price constraint implies that $W_n(H)$ must be at least as large as $W^*(H)$ which means that next period's housing price is C. The function $V^*(W)$ is defined on the interval $[W^*(H_0), W(1)]$ by (27) and (35).

We have now defined the policy functions and value function for all possible states, so this completes the definition of our candidate equilibrium $\mathcal{E}(W^*)$. Our conjectured equilibrium is the candidate equilibrium $\mathcal{E}(W^*)$ associated with the threshold wealth function W^* that satisfies a particular property. This is that for all housing levels $H \in [H_0, H^s)$, when the community has wealth $W^*(H)$, the residents are indifferent between choosing the equilibrium policies and the wealth accumulation policies $(W_n(H), H)$.[26] Our next result verifies that, if the function $V^*(W)$ associated with this threshold wealth function is strictly concave, this "guess" is indeed an equilibrium.

Theorem *Suppose that Assumptions 1-3 are satisfied. Let $W^* \in \Psi$ and let $\mathcal{E}(W^*)$ be the associated candidate equilibrium. Suppose that (i) the function $V^*(W)$ defined on the interval $[W^*(H_0), \mathcal{W}(1)]$ by (27) and (35) is strictly concave and (ii) for all $H \in [H_0, H^s)$*

$$V^*(W^*(H)) = (1-\mu)\left[\mathcal{P}(H, W_n(H), W^*(H)) + \frac{\underline{u}}{1-\beta}\right] + \mu\left[S\left(H\right) + \frac{W^*(H) - \beta W_n(H)}{H} + \beta V^*(W_n(H))\right]. \tag{37}$$

Then, $\mathcal{E}(W^*)$ is an equilibrium.

In light of this result, we will refer to a threshold wealth function that satisfies the conditions of the Theorem as an *equilibrium threshold wealth function*. To interpret the indifference condition (37), note that $V^*(W^*(H))$ (which is defined in (35)) represents the continuation payoff from the equilibrium policies when the initial state (g, b, H) is such that the community's wealth is $W^*(H)$. The expression on the right hand side represents the continuation payoff from building wealth up to $W_n(H)$, holding constant housing at H. The proof of the Theorem can be found in Appendix 2. The main task lies in establishing that the policy rules defined above are optimal for the residents in the sense of solving problem (8) when the function $V^*(W)$ is strictly concave and (37) holds. This is not a straightforward task because the choice problems faced by residents are not always well-behaved and the optimal solutions are sometimes corner solutions.

[26] By the wealth accumulation policies $(W_n(H), H)$, we mean the policies that would keep housing constant, provide the public good efficiently, and result in a wealth level $W_n(H)$ next period.

5.4 Existence of equilibrium

The Theorem leaves open the question of whether there exists an equilibrium threshold wealth function. While we do not have an analytical proof that such a function must exist, we have been able to find equilibrium threshold wealth functions numerically for specific parameterizations of the model. Indeed, we have considered a vast number of such parameterizations and in almost all those satisfying our assumptions we have found an equilibrium threshold wealth function. Specifically, we have studied 20878 different parameterizations under which there exist an interval of initial housing levels H_0 that satisfy Assumptions 1-2. For over 95% of these, there exists an equilibrium threshold wealth function for every initial housing level in the interval. A detailed account of our numerical analysis of the model can be found in Appendix 4.

6 Community development by public wealth accumulation

This section describes how the community develops in the equilibrium found in the previous section and identifies the circumstances under which equilibrium involves development by wealth accumulation. This is defined as arising whenever the community develops beyond the size that could be supported by its initial wealth. More formally, equilibrium involves development by wealth accumulation if the community's housing stock grows larger than $\max\{H_0, \mathcal{H}(W_0)\}$.[27] The way the community develops depends on its initial wealth W_0. There are three ranges - high, medium, and low - that are associated with distinct patterns of development. Development by wealth accumulation occurs in the medium and low ranges.

6.1 High initial wealth

When W_0 exceeds $\mathcal{W}(H^s)$, the initial residents' optimal plan is time consistent and the equilibrium outcome is exactly as under this plan. Thus, in the initial period, residents invest in the public good and the market provides new construction. The increase in the stock of the public good is

[27] For readers who have skipped Section 5, it will be useful to bear in mind the following definitions and concepts. First, W is the community's wealth, defined as $cg - (1+\rho)b$, and W_0 is its initial wealth. Second, $g^o(H)$ is the efficient level of the public good for a community of size H, as defined in (11). Third, $\mathcal{H}(W)$ is the largest population the community can attract with wealth W if it provides the efficient level of the public good and finances it so as to keep its wealth constant. It is formally defined in (17). Fourth, $\mathcal{W}(H)$ is the inverse of $\mathcal{H}(W)$ and is therefore the wealth the community needs to attract a population of size H if it provides the efficient level of the public good and finances it so as to keep its wealth constant. Fifth, H^s is the smallest housing level H such that, in the commitment solution, if the community has wealth less than or equal to $\mathcal{W}(H)$, residents have no incentive to accumulate wealth to attract more residents. It is formally defined in (18). Sixth, $W^*(H)$ is the threshold wealth level associated with housing level $H \in [H_0, H^s)$: new construction occurs if W is at least as large as $W^*(H)$ and there is wealth accumulation and no new construction if W is less than $W^*(H)$. This threshold wealth function is the key object determined in equilibrium.

financed entirely with debt, which keeps the community's wealth constant. Thereafter, there is no further development.[28]

Proposition 7 *Suppose that Assumptions 1-3 are satisfied. Let W^* be an equilibrium threshold wealth function and let $\mathcal{E}(W^*)$ be the associated equilibrium. If $W_0 \geq \mathcal{W}(H^s)$, then, in this equilibrium, in period 0, the community invests in $g^o(\mathcal{H}(W_0)) - g_0$ units of the public good and the market provides $\mathcal{H}(W_0) - H_0$ new houses. The community finances investment so as to keep its wealth constant, meaning that all but $c\delta g^o(\mathcal{H}(W_0))$ is financed with debt. Thereafter, the public good is maintained at $g^o(\mathcal{H}(W_0))$ and no more housing is provided. Taxes are set so that wealth remains at W_0. Throughout, the price of housing is C.*

In this case, the equilibrium does *not* involve development by wealth accumulation because the community just grows to a size $\mathcal{H}(W_0)$. To be sure, the community invests in the public good to service the new population, so that its public assets grow. However, the community finances this increase in public good stock solely with debt, so that its liabilities grow by the exact same amount. Accordingly, the community's public wealth remains constant.

6.2 Medium initial wealth

When W_0 is less than $\mathcal{W}(H^s)$, the initial residents' optimal plan is not time consistent, so the equilibrium outcome must differ from this plan. If W_0 is between $W^*(H_0)$ and $\mathcal{W}(H^s)$, the market provides new construction in the initial period, but less than the $\mathcal{H}(W_0) - H_0$ units provided under the path associated with high initial wealth. When the residents invest in the public good, they finance some of the investment with taxation and, as a result, the market provides less new construction. However, the next period, the community begins with a higher level of wealth. The same thing happens again in period 1: the residents finance some investment with taxes, which dampens new construction but increases the community's wealth a little further. This process keeps going indefinitely, with the community's housing and wealth levels gradually increasing. The size of the community converges to H^s asymptotically.

Proposition 8 *Suppose that Assumptions 1-3 are satisfied. Let W^* be an equilibrium threshold wealth function and let $\mathcal{E}(W^*)$ be the associated equilibrium. If $W_0 \in [W^*(H_0), \mathcal{W}(H^s))$, then, in this equilibrium, the market provides new construction in every period and the housing stock converges asymptotically to H^s. In all periods, the community provides the efficient level of the public*

[28] The proofs of Propositions 7, 8, and 9 can be found in Appendix 3.

good and finances some of the increase in stock with taxes. The community's wealth increases, converging asymptotically to $\mathcal{W}(H^s)$. Throughout, the price of housing is C.

In this case, the equilibrium involves development by wealth accumulation because the community grows to a size $H^s = \mathcal{H}(\mathcal{W}(H^s))$ which exceeds $\mathcal{H}(W_0)$. To illustrate how the community develops, we compute our equilibrium for a particular example. The public good benefit function has the form $B(z) = B_0 z^\sigma/\sigma$ for some $\sigma \in (0,1)$ and the parameters have the following values:

Parameter	$\bar{\theta}$	β	μ	δ	σ	B_0	α	C	c	$\underline{u}$
Value	1	$\frac{1}{1.06}$	.96	.1	.5	.34	.6	20	1	0

The results are described in Figure 1.

The top panel describes the equilibrium sequence of wealth and housing levels $\langle W_t, H_t \rangle_{t=0}^{\infty}$. Wealth is measured on the vertical axis, and housing on the horizontal. The left most dot is (W_0, H_0), the next one is (W_1, H_1), etc, etc. The line in this panel describes the equilibrium threshold wealth function $W^*(H)$, so the position of (W_0, H_0) implies that W_0 exceeds $W^*(H_0)$. In all periods, residents choose a level of wealth to carry forward equal to the threshold level associated with the new housing stock, so that (W_1, H_1) and the dots that follow all lie on this line (i.e., $(W_t, H_t) = (W^*(H_t), H_t)$). In this example, $\mathcal{H}(W_0)$ is equal to 0.435, so that it takes the community six periods to exceed the housing level that it could attract with its initial wealth. The second panel describes the evolution of the public good, taxes, and debt. The public good increases steadily as the population increases. Taxes increase, albeit very little, while debt grows. The growth in debt parallels the growth in the public good. The third panel illustrates how the community is building wealth. The upper line describes the increase in the community's public good stock, $g_t - g_{t-1}$, and the bottom line describes the increase in public wealth, $W_t - W_{t-1}$. Given that the difference between the increase in public good stock and the increase in wealth is the increase in debt, the difference between the two lines illustrates how much of the increase in the public good is paid for with debt. While most of the increase is financed by debt, a proportion is financed by taxes which is what allows wealth to build. Given that taxes are essentially constant over the course of development, this tax finance is driven by the increase in tax revenues stemming from a larger tax base. Finally, the bottom panel illustrates how the public good surplus available from living in the community evolves over time. The key point to note is that as the community's wealth increases, the surplus increases. This reflects the fiscal externality and explains why increasing numbers of potential residents choose to live in the community.

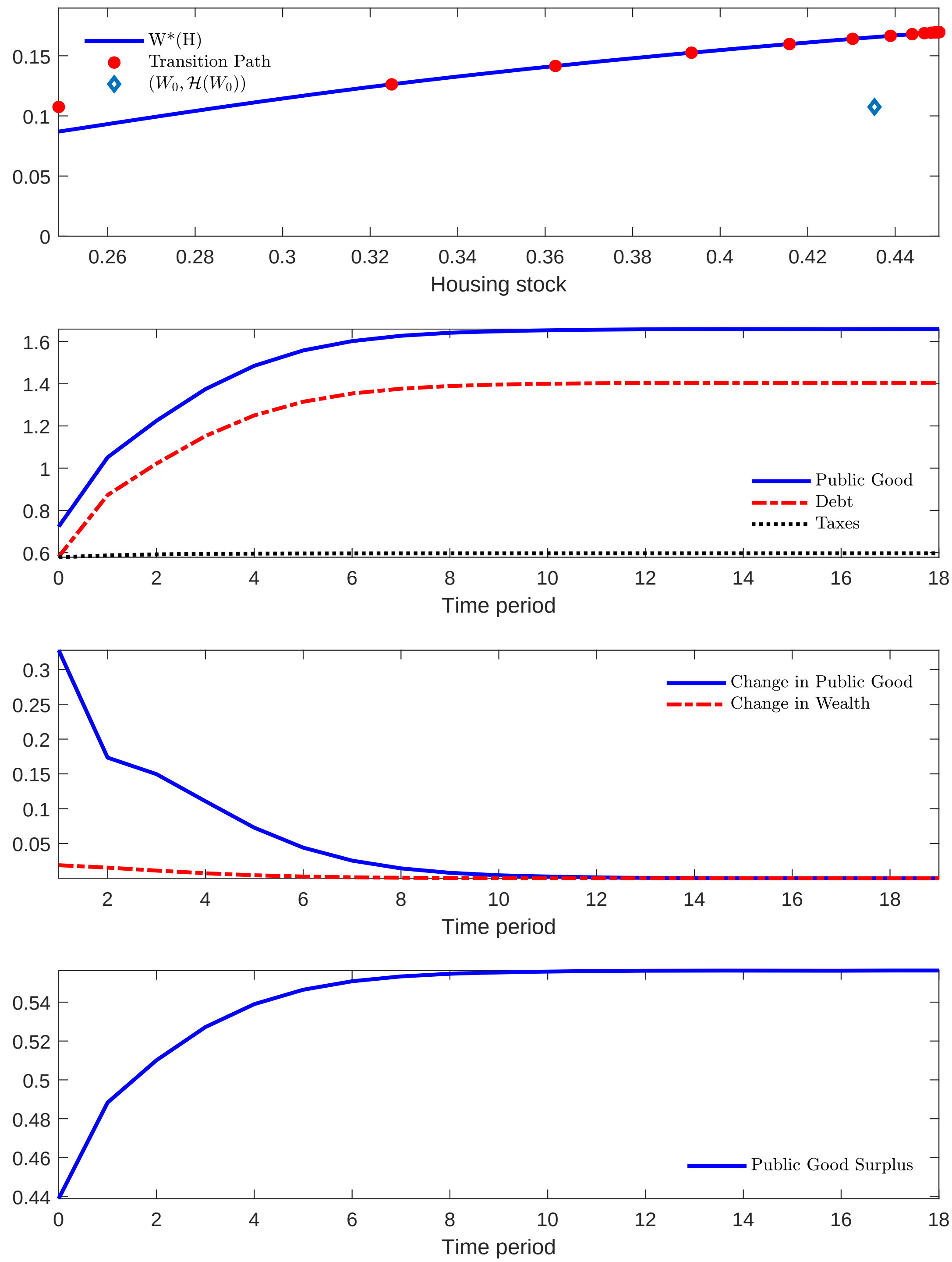

Figure 1: Development with medium initial wealth

6.3 Low initial wealth

If W_0 is smaller than $W^*(H_0)$, no new construction occurs in the initial period. Rather, the residents simply raise taxes to build up wealth. In the next period, new construction gets underway. Depending on the amount of accumulation, there can either be a single period of new construction and the housing stock jumps to H^s, or there can be new construction in every period and gradual convergence to H^s.

Proposition 9 *Suppose that Assumptions 1-3 are satisfied. Let W^* be an equilibrium threshold wealth function and let $\mathcal{E}(W^*)$ be the associated equilibrium. If $W_0 < W^*(H_0)$, then, in this equilibrium, the market provides no new construction in period 0 and the community raises taxes to build wealth to a level $W_n(H_0) \leq \mathcal{W}(H^s)$. If $W_n(H_0) < \mathcal{W}(H^s)$, the market provides new construction in every subsequent period, the housing stock converges asymptotically to H^s, and the community's wealth converges asymptotically to $\mathcal{W}(H^s)$. If $W_n(H_0) = \mathcal{W}(H^s)$, the market provides $H^s - H_0$ new houses in period 1, no new houses in subsequent periods, and wealth remains at $\mathcal{W}(H^s)$. In ether case, in all periods the community provides the efficient level of the public good. The price of housing is less than C in period 0 and C thereafter.*

In this case, the equilibrium again involves development by wealth accumulation because the community grows to size $\mathcal{H}(\mathcal{W}(H^s))$. Figure 2 illustrates how the community develops in this case. Thr right panels illustrate a case in which $W_n(H_0)$ is less than $\mathcal{W}(H^s)$ and the left panels illustrates a case in which $W_n(H_0)$ is equal to $\mathcal{W}(H^s)$. We get both cases with the same parameterization by simply varying the initial housing stock. A higher housing stock means a greater population over which to spread the costs of accumulation and that yields a higher $W_n(H_0)$. Notice that, after the initial period of accumulation, the path of development is gradual in the right panels and rapid in the left panels. Aside from this, the basic dynamics of policies in the two cases are similar.

6.4 The incentives underlying wealth accumulation

Propositions 7, 8, and 9 imply that equilibrium involves development by wealth accumulation if and only if W_0 is less than $\mathcal{W}(H^s)$.[29] It is worth understanding the incentives driving the accumulation that takes place. Interestingly, when new construction is occuring and residents are financing some of the public good investment with taxes, they are not choosing to hold back current development to subsidize future development. On the contrary, they choose the largest amount

[29] Note also that Assumption 2 is guaranteeing that the community's initial housing stock H_0 is less than H^s. If H_0 were greater than or equal to H^s, equilibrium would not involve development by wealth accumulation.

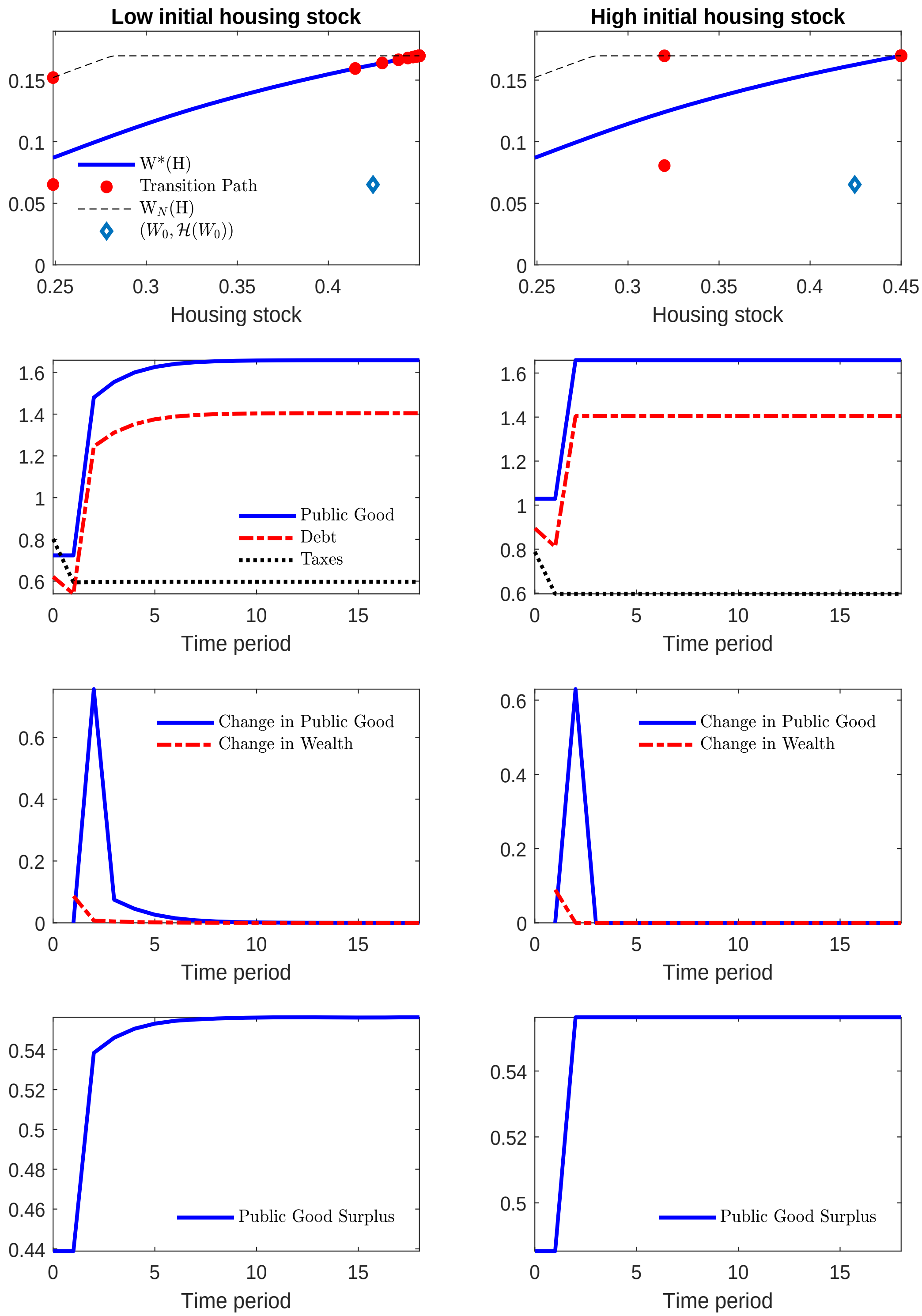

Figure 2: Development with low initial wealth

of development they can and it is the off-equilibrium path behavior of future residents that forces accumulation.

To understand this, suppose we are in the medium wealth case and consider what happens in the initial period. The equilibrium policy involves the market providing a level of new construction equal to $H_c(W_0) - H_0$ and residents increasing wealth to the threshold level associated with the new housing stock $W^*(H_c(W_0))$ where $H_c(W_0)$ is defined in (31). Note first that, by choosing higher taxes, the residents could increase the wealth level carried forward to some level beyond $W^*(H_c(W_0))$ at the cost of reduced development in the initial period. Development is reduced because the higher taxes deter some potential residents from purchasing a house in the community. The proof of the Theorem shows that residents have no incentive to hold back development in this way. Intuitively, the residents are already providing next period's residents a wealth level sufficient to deter them from stopping development and accumulating more wealth. Since next period's residents have a lower cost of accumulation, current residents have no incentive to accumulate more than this threshold level.

The more subtle issue is whether residents could attract more development by choosing lower taxes? The answer is no. Suppose they were to choose a small tax reduction with the aim of carrying forward a wealth level W' marginally lower than the threshold $W^*(H_c(W_0))$ and attracting marginally more potential residents. Then, in response to this deviation from equilibrium play, next period's residents would choose to raise taxes to accumulate wealth and no development would take place. The price of housing in the next period would fall discontinuously below the construction cost C in response to this wealth accumulation initiative. This fall would necessitate a reduction in the current price of housing in order to continue to attract a population of size $H_c(W_0)$. Intuitively, potential residents would need to be compensated for the reduction in the future value of their homes. But this would drive the current price of housing below the construction cost C and would mean that the market would not supply any new construction. Thus, a marginal reduction in taxes would mean that residents could no longer attract a population of $H_c(W_0)$ - let alone a larger population. A bigger reduction in taxes would not help, because the lower level of wealth carried forward would be perfectly capitalized in next period's price of housing.

This discussion reveals an interesting non-monotonic relationship between debt-financed tax reductions and potential residents' desire to live in the community. Debt-financed tax reductions are attractive to potential residents provided that development continues in the next period. For as long as this is the case, the future value of potential residents' homes remains at the construction cost C. A tax reduction then simply creates an increase in public good surplus which increases the desire of potential residents to live in the community (see (5)). Once the tax reduction is high enough to drive next period's wealth below the threshold, development stops in the next period and a wealth accumulation initiative is triggered. This causes a discontinuous reduction in the future value of potential residents' homes and potential residents' desire to live in the community. Further tax reductions beyond this point simply cause an even bigger reduction in future home values via the capitalization of higher debt.[30]

It is in the initial period in the low wealth case, that the residents may choose to hold back development. In equilibrium, there is no new construction and the residents build the community's wealth to $W_n(H_0)$ by raising taxes, paving the way for development in period 1. As in the commitment case, this occurs even when development is feasible. In particular, if W_0 is smaller than but sufficiently close to $W^*(H_0)$, there exists a housing level H' larger than H_0 such that if the residents provided a public good level $g^o(H')$ and financed it in such a way as to increase wealth to $W^*(H')$, the market would respond by providing $H' - H_0$ units of new construction.[31] These alternative policies offer a higher payoff in the initial period than the equilibrium policies as they involve both a higher housing price and more development. They also allow development to continue in the next period. However, they offer less payoff in the future, because the wealth level $W_n(H_0)$ exceeds $W^*(H')$ and permits more development in the next period.

6.5 The significance of congestibility

Development by wealth accumulation occurs when the community's initial wealth is less than $\mathcal{W}(H^s)$. This raises the question of what determines $\mathcal{W}(H^s)$? The key determinant is the congestibility of the public good (α). As is clear from (15), the size of α determines the extent of the positive fiscal externality arising from sharing the costs of public good provision. The case of $\alpha = 1$ is of particular interest because there is no positive fiscal externality arising from the public good. Thus, the incentive for accumulation arises only when the fiscal externality created by the collective ownership of community assets is positive, which requires that the community has negative initial wealth. Intuitively, when the community has negative wealth, it has a debt burden

[30] This capitalization logic also explains why residents have no incentive to implement debt-financed transfers. In addition, it explains why a transversality condition on the community's debt is not needed.

[31] We know that $\mathcal{P}(H_0, W^*(H_0), W^*(H_0))$ is larger than $\mathcal{P}(H_0, \mathcal{W}(H_0), \mathcal{W}(H_0))$ which is equal to C. Thus, for W_0 smaller than but sufficiently close to $W^*(H_0)$, it must be the case that $\mathcal{P}(H_0, W^*(H_0), W_0)$ is larger than C. Moreover, W_0 is less than $\mathcal{W}(H^s)$ which equals $W^*(H^s)$. It follows that $\mathcal{P}(H^s, W^*(H^s), W_0)$ is less than C. Thus, by continuity, there exist $H' \in (H_0, H^s)$ such that $\mathcal{P}(H', W^*(H'), W_0) = C$. Such a H' has the properties discussed in the text.

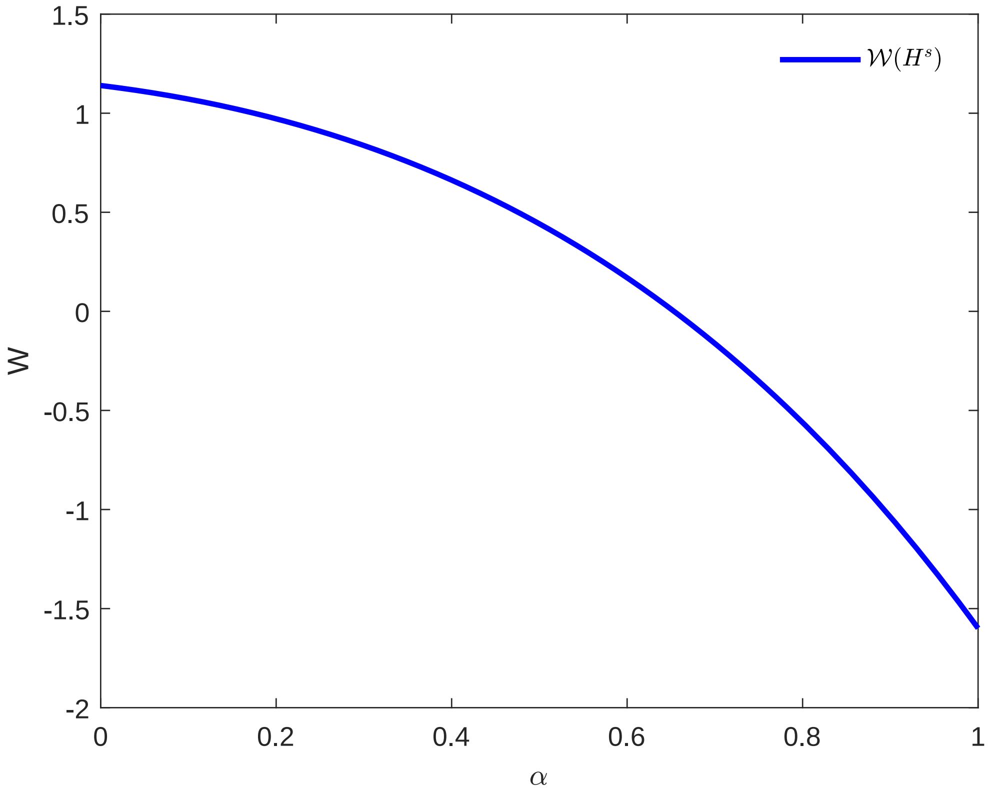

Figure 3: The critical wealth level

and residents would like to attract more residents to share the burden of servicing it. Formally, this can be seen by computing $\mathcal{W}(H^s)$ which equals

$$\mathcal{W}(H^s) = -\frac{H^s\left((1-H^s)\,\overline{\theta} + S - C(1-\beta) - \underline{u}\right)}{1-\beta} < 0, \tag{38}$$

where S is the optimized public good surplus (which is independent of H when $\alpha = 1$). At the other extreme, we have the case of $\alpha = 0$ in which the public good is a pure public good. This creates the largest positive fiscal externality from cost sharing and the strongest incentives to attract new residents. In this case, using (14), (15), and (17), it is easy to show that

$$\mathcal{W}(H^o) = \frac{cg^o(H^o)(1-\beta(1-\delta))}{1-\beta}. \tag{39}$$

Recall that $\mathcal{W}(H^s)$ approaches $\mathcal{W}(H^o)$ as μ converges to 1, so that for sufficiently large μ, $\mathcal{W}(H^s)$ exceeds $cg^o(H^o)$. Thus, even if the community begins with the optimal level of public good for its initial population and no debt, the equilibrium will involve wealth accumulation.

Figure 3 generalizes these observations by graphing $\mathcal{W}(H^s)$ as a function of α holding constant

H^s. The latter is achieved by varying the utility obtained from living outside the community. Parameters other than α and $\underline{u}$ are set at their benchmark values. Bearing in mind that community development by wealth accumulation occurs if and only if W_0 is less than $\mathcal{W}(H^s)$, the Figure nicely illustrates the role played by the cost-sharing fiscal externality in inducing such behavior. The more important the cost-sharing externality, the larger the set of initial wealth levels for which development by wealth accumulation arises.

7 The optimality of development by wealth accumulation

This section considers how well community development by wealth accumulation works from a normative perspective. Comparing Proposition 1 with Propositions 8 and 9, yields the following conclusion.[32]

Proposition 10 *Suppose that Assumptions 1-3 are satisfied. Let $W^* \in \Psi$ be an equilibrium threshold wealth function and let $\mathcal{E}(W^*)$ be the associated equilibrium. If $W_0 < \mathcal{W}(H^s)$ the long-run size of the community in equilibrium will be smaller than optimal. In addition, the equilibrium exhibits delay because development occurs after period* 0.

This proposition reveals that there are two problems with development by wealth accumulation: it does not get the community to its optimal size and, such development as does occur, proceeds too slowly. To understand the first problem, imagine that the community were at its steady state with wealth $\mathcal{W}(H^s)$ and housing level H^s. Why would the residents not accumulate a little more wealth which would increase the future size of the community a little more? Residents must weigh the benefit of increasing the future size of the community via the fiscal externality against the tax cost. The equilibrium increase in the future population would be $\mathcal{H}'(\mathcal{W}(H^s))$. The marginal cost of accumulation is $1/H^s$. As pointed out after Proposition 4, at housing level H^s, the marginal benefit of increasing the population is just equal to the marginal cost. Thus, given the concavity of $\mathcal{H}'(W)$, there is no incentive to accumulate.

Why is development delayed? To shed light on this, it is instructive to consider the policy choices in the initial period and analyze why residents prefer the equilibrium choices to those involving less delay. In the medium initial wealth case, equilibrium policies are such that the community's wealth and housing increase but to levels smaller than $(\mathcal{W}(H^s), H^s)$. Since the community's wealth and housing will eventually increase to $(\mathcal{W}(H^s), H^s)$, why do current residents

[32] To understand this result and the discussion to follow, the reader will need to recall that H^o is the optimal housing level defined in (14) and to bear in mind that $H^s \in (H_0, H^o)$.

not prefer to just jump directly to $(\mathcal{W}(H^s), H^s)$? The reason is that raising wealth to this level would require higher taxes and, given these taxes, the market would not provide $H^s - H_0$ new homes. As noted in Section 6.4, residents are choosing the maximum level of development they can.

In the low initial wealth case, the equilibrium involves no new construction in period 0 and the community's wealth growing to $W_n(H_0)$. Development is delayed at least one period. Again, potential residents do not want to join the community given its high taxes. If $W_n(H_0)$ is equal to $\mathcal{W}(H^s)$, then all development occurs in period 1 and there is no further delay. If $W_n(H_0)$ is less than $\mathcal{W}(H^s)$, then there is further delay as the housing stock gradually grows to H^s. The initial residents could instead choose to increase the community's wealth all the way to $\mathcal{W}(H^s)$, reducing delay to just one period. However, the tax burden of the increase in wealth must be bourne solely by the period 0 residents, whereas the benefits of the higher wealth are shared by the new residents. Thus, this will not be an attractive strategy, unless H_0 is quite close to H^s.[33]

Further insight into the forces driving these distortions can be obtained by considering the limit as μ tends to 1 in which case residents know that they will remain in the community forever. From (18), we see that H^s tends to H^o. It follows that it is population turnover that is responsible for the fact that the community will be too small in the long run. In addition, it can be shown that the range of values of H_0 for which $W_n(H_0)$ equals $\mathcal{W}(H^s)$ vanishes. This means that if W_0 is below $\mathcal{W}(H^o)$, the community will gradually grow towards the optimal size H^o. It follows that population turnover is not responsible for the fact that development is too slow. Gradual development reflects the fact that the costs of accumulation are lower when the population is larger. In this way, development is necessary to spur future development.

8 Discussion

This section discusses the empirical plausibility of the idea of community development by public wealth accumulation and identifies some open questions. It also draws out some broader lessons of the model.

[33] A sufficient condition for $W_n(H_0)$ to be less than $\mathcal{W}(H^s)$ is that H_0 is less than $(1-\beta)\, H^s/\mu\,(1-\mu\beta)$. Note that for sufficiently large μ, this condition is satisfied for any H_0 less than H^s.

8.1 Plausibility

The logic underlying development by wealth accumulation rests on two basic hypotheses. The first is that potential residents will be influenced in their location decisions by a community's wealth. This is necessary to create the fiscal externality. The second is that residents will be willing to support higher taxes to improve their community's wealth in order to attract more potential residents. This is necessary for the fiscal externality to shape policy choices.

The economics behind the first hypothesis is that higher public wealth allows a community to provide more public good surplus and this is what draws in potential residents. The capitalization literature provides empirical support for the idea that residents are attracted to communities by the public good surplus they offer. This literature leverages the theoretical idea that, if potential residents value public services and dislike taxes, then in communities which provide higher public services (controlling for taxes) or lower taxes (controlling for services), there must be some compensating differential to maintain locational equilibrium if space in these communities is scarce.[34] Typically, the compensating differential focused on is higher housing prices, but wage differentials are also a theoretical possibility in settings where firms choose where to produce.[35]

A particularly relevant example of this style of work is Gyourko and Tracy (1991) who study capitalization of public sector surplus into wages and housing prices. Their underlying theoretical framework is one in which homogeneous worker/residents and firms compete for scarce sites across jurisdictions. Jurisdictions are characterized by natural amenities, public service levels, and tax rates. Wages and gross of tax land prices (which are measured by housing expenditures) adjust to make workers and firms indifferent to their location. This set-up motivates regressing wages and housing expenditures against measures of public services (police, fire, health, and education), taxes, and community-level natural amenities (weather, pollution, proximity to oceans and lakes, etc). Gyourko and Tracy's results support the idea that potential residents value public services and dislike taxes and suggest that fiscal variables are nearly as important to worker/residents as natural amenities.

As stressed by Gyourko and Tracy, public service levels and taxes differ from natural amenities in that they are under the control of local authorities. This raises the question of what is allowing

[34] Recall that in the model of this paper, there is sufficient space to accomodate those who are open to living in the community. Moreover, the supply of housing is perfectly elastic once price reaches construction cost. Thus, after this point, higher public good surplus is dissipated by development rather than housing price increases.

[35] The bulk of the capitalization literature focuses on the impact of property taxes and school quality on housing prices. See Ross and Yinger (1999) and Nguyen-Hoang and Yinger (2011) for surveys.

jurisdictions to offer higher public sector surplus to their residents. Our model suggests that it is higher community wealth.[36] Intuitively, a community with better school facilities and community health centers, more modern fire stations, fire trucks, police cars, etc, and lower public debt, is in a stronger position to offer higher public sector surplus. This suggests that there should be a positive relationship between a community's wealth and the public sector surplus that it offers. However, we are not aware of any empirical studies that analyze this relationship.

Turning to the second hypothesis, in the model, a community's wealth can be improved in two ways: tax-financed public investment or reducing debt. In reality, another way is to finance public investment by reducing non-capital spending on other services. Regarding public investment, the practical literature on community development documents numerous examples of communities improving their public assets with an eye to attracting more residents and/or businesses (see, for example, Phillips and Pittman 2015). Communities undertake projects to revitalize their downtowns and waterfronts, improve their infrastructure, and build museums and parks. They do this both to benefit their residents and also to attract more development.[37] The rationale for attracting more development may vary communities, but all that matters for this paper's argument is that there be some rationale.[38]

Of course, it is not clear that such investment projects are financed by higher taxes on current residents or by reducing spending on other services. Indeed, many will be primarily financed by grants from state or federal government.[39] These grants will permit the community's wealth to increase and should attract potential residents via the fiscal externality, but this is a different mechanism than the locally-generated development studied here. Obviously, residents will prefer

[36] A possibility suggested by Gyourko and Tracy is that communities offering higher public sector surplus have weaker public sector unions or less corrupt politicians. While such differences across jurisdictions are very plausible, it seems like community wealth must matter as well.

[37] Larger communities also routinely put together packages to attract or retain specific businesses (see, for example, Garcia-Mila and MacGuire 2002 and Greenstone and Moretti 2003). These deals include tax breaks, low-cost or free land, or targeted infrastructure investments. To the extent that the targeted business creates agglomeration economies that benefit the community at large (see Geenstone, Hornbeck, and Moretti 2010 for evidence), these packages have a similar flavor to the public wealth accumulation studied here. The idea is that current residents undertake an investment that spurs future development which benefits the community via an externality. Indeed, if the agglomeration economy is sufficiently large, then such an investment should be able to spur development even if financed by debt. In practice, financing is typically shared with state governments. Moreover, tax breaks to attract a new business do not require changing current taxes or services to finance, which reduces the up-front cost to current residents. An interesting subject for further research is to consider the offering of such policies to attract businesses in a dynamic political economy model like the one studied here.

[38] This is a point worth emphasizing. While our model incorporates a positive fiscal externality created by the sharing of the public good, any positive externality from additional residents would motivate wealth accumulation. As reviewed in Duranton and Puga (2004), the urban economics literature identifies a number of different reasons for agglomeration economies.

[39] Another possibility that arises in practice is that development-related community investments are financed by private contributions from groups of affluent local citizens. See, for example, Goldstein (2017) and Sullivan (2018).

their projects to be financed by higher levels of government than by taxation, so it is natural that their elected representatives seek out such grants. Moreover, as we argue below, the motivation for such inter-governmental grants could be precisely that communities will under-accumulate the wealth they need to develop if left unaided.

Regarding the strategy of improving community wealth by reducing debt, we are not aware of any studies that shed light on the prevalence of this. It might be useful to study communities in poor fiscal health due to negative economic shocks and/or large unfunded public employee pension liabilities and document how they respond to their situations. As suggested by the model, these type of communities should be the most likely to levy higher taxes on residents or reduce spending on other services to improve their fiscal health. Again, however, one would expect state governments to play a role in supporting struggling communities, which may obscure the picture empirically.

To sum up, we see two open empirical questions raised by the idea of development by wealth accumulation. First, is it the case that wealthier communities offer higher public sector surplus? Second, how common is it for communities to raise taxes or reduce non-capital spending on other services in order to finance new public investments or pay down debt? Further evidence on these questions would be helpful. It would also be interesting to try and directly test the idea of development by wealth accumulation. The central prediction is that development should be positively related to a community's wealth. A basic difficulty in testing this lies in measuring the value of a community's public assets. A further problem would be in adequately controlling for all the other determinants of development.[40]

8.2 Broader lessons

The most basic lesson of the model is that the collective ownership of a community's assets and liabilities creates a fiscal externality. This emerges organically from the assumption that the community provides a durable public good and can finance investment with debt. One point not emphasized in the analysis so far is that this fiscal externality can lead a well-endowed community to develop too much. Proposition 7 implies that if initial wealth W_0 exceeds $\mathcal{W}(H^o)$, the community will be too large. The excess entry into the community that arises in this case is analogous to the excess entry familiar in the standard common pool problem and is one way of thinking about the

[40] In a well-known study of the determinants of U.S. city growth, Glaeser, Scheinkman, and Shleifer (1995) find that 1960 debt levels (holding revenue constant) are positively associated with subsequent population growth over the period 1960-1990. However, their study does not include information on city assets.

problem of urban sprawl (Nechyba and Walsh 2004). Here the negative fiscal externality created by the collective ownership of community wealth offsets the positive fiscal externality that arises from the cost-sharing of the public good. Residents might like to keep the new entrants out, but there is no way they can do so with the available policy instruments.

The fiscal externality created by collective ownership is relevant for the long-standing policy debate about the desirability of decentralizing fiscal decisions to local communities. On the negative side, it is another reason for believing that a decentralized system in which local governments choose fiscal policies and the free market determines the allocation of households across communities is not likely to produce optimally-sized communities. On the positive side, the fiscal externality does provide a simple mechanism by which decentralized communities can influence their size - at least if they wish to grow. While there are good reasons to suppose that this mechanism will not allow communities to achieve fully efficient sizes, it should allow some development.

The fact that residents will under-accumulate wealth may also have implications for inter-governmental grants. As noted in the previous sub-section, many projects chosen by local communities are financed partially by higher levels of government. The traditional way to motivate such grants is that they account for inter-juridictional spillovers in the benefits of local public goods. This model suggests that they may be a way of allowing communities to build the wealth necessary for them to grow to optimal size. To further analyze this and the general question of the desirability of decentralization, it will be important to build dynamic general equilibrium models featuring multiple communities providing durable public goods with taxes and debt.

The standard way to deal with an externality is to impose a Pigouvian tax or subsidy to internalize it. For the fiscal externality created by the collective ownership of assets and liabilities, a tax or subsidy on new construction would be appropriate. A tax could be levied when the community has positive wealth and there is no off-setting fiscal externality or agglomeration economy. A subsidy could be granted when the community has negative wealth or when it has positive wealth, but there is an off-setting cost-sharing or agglomeration economy. In practice, subsidies for new construction do not seem to arise which could reflect practical (e.g., informational) and/or legal barriers to their use.[41] Taxes are also not common, but zoning and impact fees are certainly ways communities control entry. While in principle, a tax or subsidy on new construction would allow residents to price fiscal externalities appropriately, residents' policy choices will be driven

[41] An exception would be support offered for the provision of affordable (i.e., low income) housing through the Community Development Block Grant Program operated by the U.S. Department of Housing and Urban Development.

by concern over the price of their properties and the associated revenues or costs. Furthermore, time consistency issues arise in a way that relates to the classic literature on monopoly pricing of durable goods without commitment (see, for example, Bulow 1982 and Stokey 1981). These issues give rise to interesting development dynamics and it is by no means obvious that social welfare or even the welfare of current residents is higher when these instrument are available.[42]

9 Conclusion

This paper presents and analyzes a novel political economy model of community development. The main lesson from the analysis concerns the potential role of public wealth accumulation in development. The key observation is that the collective ownership of a community's assets and liabilities create a fiscal externality. This fiscal externality is positive when a community has negative public wealth and negative when a community has positive wealth. When making fiscal decisions, residents will anticipate their impact on this fiscal externality. A community with negative public wealth, may want to increase it to benefit from the positive externality. A community with positive wealth may want to increase it to attract residents because of other positive fiscal externalities or agglomeration economies. If such development occurs, future residents may have an incentive to engage in more of it. This is because the cost of further wealth increases can be bourne by a larger group of residents. In this way, the community can develop fueled by public wealth accumulation.

The model reveals the forces that shape such development and sheds light on its normative properties. Development by wealth accumulation arises if the community starts out small enough and with low enough initial wealth. Several patterns of development are possible, depending on the community's initial conditions. With a medium level of initial wealth, development will be gradual and continual, with the community converging asymptotically to a steady state level. With a low level of initial wealth, the community will have a period of accumulation which precedes development. The subsequent development can be gradual or rapid depending on the extent of initial accumulation. Development by wealth accumulation will not take the community to an efficient size and will proceed too slowly. Nonetheless, the mechanism allows even very poorly endowed communities to develop to some extent.

In addition to revealing the idea of development by wealth accumulation, the model presented here provides a platform for further theoretical investigation of community development.[43] There

[42] We are currently working on this problem.

[43] A variant of this model could also be used to study the dynamic development of clubs (country clubs, golf

are many questions to be addressed and we have pointed out some along the way. Understanding what would happen if residents could use additional policy instruments such as zoning or a tax/subsidy on new construction is obviously of significant interest. It would also be interesting to extend the model to understand declining communities and the role played by local governments in managing or combating decline. Finally, constructing a dynamic general equilibrium model with multiple communities providing durable local public goods with taxes and debt would allow a number of important issues to be addressed.

clubs, social clubs, hobby clubs, community associations, etc). All that would be necessary would be to remove housing. In this conception, a club would be characterized by its membership size, its stock of public good, and its debt. In each period, the club invests in the public good and finances this either by a tax on members or a debt issue. Club decisions are made collectively by current members. The only difference between this and our model, is that individuals do not have to buy a house to join the club. They just have to pay the tax (i.e., membership fee) to benefit. Any existing member can avoid the tax by just leaving the club. In particular, they do not have to sell their house. In a sense, this set-up is simpler than the model considered here, because there is no housing market. On the other hand, the lack of durable housing means there is nothing to anchor club size: in particular, it could shrink.

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